## Immersion Math 30 Practice Problems

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Step 2. Write the equivalent compound inequality.
$|u|>a$ is equivalent to
$u<-a$ or $u>a$
$|u| \geq a \quad$ is equivalent to $\quad u \leq-a$ or $u \geq a$

Step 3. Solve the compound inequality.
Step 4. Graph the solution
Step 5. Write the solution using interval notation

## REVIEW EXERCISES

### 2.1 Use a General Strategy to Solve Linear Equations

## Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, determine whether each number is a solution to the equation.
496. $10 x-1=5 x, x=\frac{1}{5}$
497. $-12 n+5=8 n, n=-\frac{5}{4}$

In the following exercises, solve each linear equation.
498. $6(x+6)=24$
499. $-(s+4)=18$
500. $23-3(y-7)=8$
501. $\frac{1}{3}(6 m+21)=m-7$
502. $4(3.5 y+0.25)=365$
503. $0.25(q-8)=0.1(q+7)$
504. $8(r-2)=6(r+10)$
505. $5+7(2-5 x)=2(9 x+1)-(13 x-57)$
506. $(9 n+5)-(3 n-7)=20-(4 n-2)$
507. $2[-16+5(8 k-6)]=8(3-4 k)-32$

## Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.
508. $17 y-3(4-2 y)=11(y-1)+12 y-1$
509. $9 u+32=15(u-4)-3(2 u+21)$
510. $-8(7 m+4)=-6(8 m+9)$

Solve Equations with Fraction or Decimal Coefficients
In the following exercises, solve each equation.
511. $\frac{2}{5} n-\frac{1}{10}=\frac{7}{10}$
512. $\frac{3}{4} a-\frac{1}{3}=\frac{1}{2} a+\frac{5}{6}$
513. $\frac{1}{2}(k+3)=\frac{1}{3}(k+16)$
514. $\frac{5 y-1}{3}+4=\frac{-8 y+4}{6}$
515. $0.8 x-0.3=0.7 x+0.2$
516. $0.10 d+0.05(d-4)=2.05$

### 2.2 Use a Problem-Solving Strategy

Use a Problem Solving Strategy for Word Problems
In the following exercises, solve using the problem solving strategy for word problems.
517. Three-fourths of the people at a concert are children. If there are 87 children, what is the total number of people at the concert?
518. There are nine saxophone players in the band. The number of saxophone players is one less than twice the number of tuba players. Find the number of tuba players.

## Solve Number Word Problems

In the following exercises, solve each number word problem.
519. The sum of a number and three is forty-one. Find the number.
522. The sum of two consecutive integers is -135 . Find the numbers.
525. Koji has $\$ 5,502$ in his savings account. This is $\$ 30$ less than six times the amount in his checking account. How much money does Koji have in his checking account?
520. One number is nine less than another. Their sum is negative twenty-seven. Find the numbers.
523. Find three consecutive even integers whose sum is 234 .

## Solve Percent Applications

In the following exercises, translate and solve.
526. What number is $67 \%$ of 250 ?

In the following exercises, solve.
529. The bill for Dino's lunch was $\$ 19.45$. He wanted to leave $20 \%$ of the total bill as a tip. How much should the tip be?
532. Angel received a raise in his annual salary from $\$ 55,400$ to $\$ 56,785$. Find the percent change.
535. Lacey bought a pair of boots on sale for $\$ 95$. The original price of the boots was $\$ 200$. Find (a) the amount of discount and (b) the discount rate. (Round to the nearest tenth of a percent, if needed.)
530. Dolores bought a crib on sale for $\$ 350$. The sale price was $40 \%$ of the original price. What was the original price of the crib?
533. Rowena's monthly gasoline bill dropped from $\$ 83.75$ last month to $\$ 56.95$ this month. Find the percent change.
536. Nga and Lauren bought a chest at a flea market for $\$ 50$. They re-finished it and then added a $350 \%$ mark-up. Find (a) the amount of the mark-up and (b) the list price.
521. One number is two more than four times another. Their sum is negative thirteen. Find the numbers.
524. Find three consecutive odd integers whose sum is 51 .
528. What percent of 125 is 150 ?
531. Jaden earns $\$ 2,680$ per month. He pays $\$ 938$ a month for rent. What percent of his monthly pay goes to rent?
534. Emmett bought a pair of shoes on sale at $40 \%$ off from an original price of $\$ 138$. Find (a) the amount of discount and (b) the sale price.

Solve Simple Interest Applications
In the following exercises, solve.
537. Winston deposited $\$ 3,294$ in a bank account with interest rate 2.6\% How much interest was earned in five years?
538. Moira borrowed $\$ 4,500$ from her grandfather to pay for her first year of college. Three years later, she repaid the $\$ 4,500$ plus $\$ 243$ interest. What was the rate of interest?

### 2.3 Solve a formula for a Specific Variable

Solve a Formula for a Specific Variable
In the following exercises, solve the formula for the specified variable.
540. Solve the formula
541. Solve the formula
$V=L W H$ for $L$.
$A=\frac{1}{2} d_{1} d_{2}$ for $d_{2}$.
543. Solve the formula
$4 x-3 y=12$ for $y$.

## Use Formulas to Solve Geometry Applications

In the following exercises, solve using a geometry formula.
544. What is the height of a triangle with area 67.5 square meters and base 9 meters?
545. The measure of the smallest angle in a right triangle is $45^{\circ}$ less than the measure of the next larger angle. Find the measures of all three angles.
548. Find the length of the missing side. Round to the nearest tenth, if necessary.

539. Jaime's refrigerator loan statement said he would pay $\$ 1,026$ in interest for a four-year loan at $13.5 \%$. How much did Jaime borrow to buy the refrigerator?
542. Solve the formula $h=48 t+\frac{1}{2} a t^{2}$ for $t$.
546. The perimeter of a triangle is 97 feet. One side of the triangle is eleven feet more than the smallest side. The third side is six feet more than twice the smallest side. Find the lengths of all sides.
549. Sergio needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is eight feet tall and Sergio has 10 feet of wire. How far from the base of the antenna can he attach the wire? Approximate to the nearest tenth, if necessary.

552. The width of a rectangle is three more than twice the length. The perimeter is 96 inches. Find the length and the width.
553. The perimeter of a triangle is 35 feet. One side of the triangle is five feet longer than the second side. The third side is three feet longer than the second side. Find the length of each side.

### 2.4 Solve Mixture and Uniform Motion Applications

## Solve Coin Word Problems

In the following exercises, solve.
554. Paulette has $\$ 140$ in $\$ 5$ and $\$ 10$ bills. The number of $\$ 10$ bills is one less than twice the number of $\$ 5$ bills. How many of each does she have?
555. Lenny has $\$ 3.69$ in pennies, dimes, and quarters. The number of pennies is three more than the number of dimes. The number of quarters is twice the number of dimes. How many of each coin does he have?

## Solve Ticket and Stamp Word Problems

In the following exercises, solve each ticket or stamp word problem.
556. Tickets for a basketball game cost $\$ 2$ for students and $\$ 5$ for adults. The number of students was three less than 10 times the number of adults. The total amount of money from ticket sales was $\$ 619$. How many of each ticket were sold?

## Solve Mixture Word Problems

## In the following exercises, solve

559. Marquese is making 10 pounds of trail mix from raisins and nuts. Raisins cost $\$ 3.45$ per pound and nuts cost $\$ 7.95$ per pound. How many pounds of raisins and how many pounds of nuts should Marquese use for the trail mix to cost him $\$ 6.96$ per pound?
560. 125 tickets were sold for the jazz band concert for a total of $\$ 1,022$. Student tickets cost $\$ 6$ each and general admission tickets cost $\$ 10$ each. How many of each kind of ticket were sold?
561. Amber wants to put tiles on the backsplash of her kitchen counters. She will need 36 square feet of tile. She will use basic tiles that cost $\$ 8$ per square foot and decorator tiles that cost $\$ 20$ per square foot. How many square feet of each tile should she use so that the overall cost of the backsplash will be $\$ 10$ per square foot?

## Solve Uniform Motion Applications

In the following exercises, solve.
562. When Gabe drives from Sacramento to Redding it takes him 2.2 hours. It takes Elsa two hours to drive the same distance. Elsa's speed is seven miles per hour faster than Gabe's speed Find Gabe's speed and Elsa's speed.
563. Louellen and Tracy met at a restaurant on the road between Chicago and Nashville. Louellen had left Chicago and drove 3.2 hours towards Nashville. Tracy had left Nashville and drove 4 hours towards Chicago, at a speed one mile per hour faster than Louellen's speed. The distance between Chicago and Nashville is 472 miles. Find Louellen's speed and Tracy's speed.
558. Yumi spent $\$ 34.15$ buying stamps. The number of $\$ 0.56$ stamps she bought was 10 less than four times the number of $\$ 0.41$ stamps. How many of each did she buy?
561. Enrique borrowed $\$ 23,500$ to buy a car. He pays his uncle 2\% interest on the $\$ 4,500$ he borrowed from him, and he pays the bank $11.5 \%$ interest on the rest. What average interest rate does he pay on the total $\$ 23,500$ ? (Round your answer to the nearest tenth of a percent.)
564. Two busses leave Amarillo at the same time. The Albuquerque bus heads west on the I-40 at a speed of 72 miles per hour, and the Oklahoma City bus heads east on the I-40 at a speed of 78 miles per hour. How many hours will it take them to be 375 miles apart?
565. Kyle rowed his boat upstream for 50 minutes. It took him 30 minutes to row back downstream. His speed going upstream is two miles per hour slower than his speed going downstream. Find Kyle's upstream and downstream speeds.
566. At 6:30, Devon left her house and rode her bike on the flat road until 7:30. Then she started riding uphill and rode until 8:00. She rode a total of 15 miles. Her speed on the flat road was three miles per hour faster than her speed going uphill. Find Devon's speed on the flat road and riding uphill.
567. Anthony drove from New York City to Baltimore, which is a distance of 192 miles. He left at 3:45 and had heavy traffic until 5:30. Traffic was light for the rest of the drive, and he arrived at 7:30. His speed in light traffic was four miles per hour more than twice his speed in heavy traffic. Find Anthony's driving speed in heavy traffic and light traffic.

### 2.5 Solve Linear Inequalities

Graph Inequalities on the Number Line
In the following exercises, graph the inequality on the number line and write in interval notation.
568. $x<-1$
569. $x \geq-2.5$
570. $x \leq \frac{5}{4}$
571. $x>2$
572. $-2<x<0$
573. $-5 \leq x<-3$
574. $0 \leq x \leq 3.5$

Solve Linear Inequalities
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.
575. $n-12 \leq 23$
576. $a+\frac{2}{3} \geq \frac{7}{12}$
577. $9 x>54$
578. $\frac{q}{-2} \geq-24$
579. $6 p>15 p-30$
580. $9 h-7(h-1) \leq 4 h-23$

## 581.

$5 n-15(4-n)<10(n-6)+10 n$
582. $\frac{3}{8} a-\frac{1}{12} a>\frac{5}{12} a+\frac{3}{4}$

Translate Words to an Inequality and Solve
In the following exercises, translate and solve. Then write the solution in interval notation and graph on the number line.
583. Five more than $z$ is at most
584. Three less than $c$ is at least
360.
586. Negative two times $a$ is no more than eight.

## Solve Applications with Linear Inequalities

In the following exercises, solve.
587. Julianne has a weekly food budget of $\$ 231$ for her family. If she plans to budget the same amount for each of the seven days of the week, what is the maximum amount she can spend on food each day?
588. Rogelio paints watercolors. He got a $\$ 100$ gift card to the art supply store and wants to use it to buy 12" $\times 16^{\prime \prime}$ canvases. Each canvas costs $\$ 10.99$. What is the maximum number of canvases he can buy with his gift card?
589. Briana has been offered a sales job in another city. The offer was for $\$ 42,500$ plus $8 \%$ of her total sales. In order to make it worth the move, Briana needs to have an annual salary of at least $\$ 66,500$. What would her total sales need to be for her to move?
590. Renee's car costs her \$195 per month plus $\$ 0.09$ per mile. How many miles can Renee drive so that her monthly car expenses are no more than $\$ 250$ ?
591. Costa is an accountant During tax season, he charges $\$ 125$ to do a simple tax return. His expenses for buying software, renting an office, and advertising are $\$ 6,000$. How many tax returns must he do if he wants to make a profit of at least $\$ 8,000$ ?
592. Jenna is planning a five-day resort vacation with three of her friends. It will cost her $\$ 279$ for airfare, $\$ 300$ for food and entertainment, and $\$ 65$ per day for her share of the hotel. She has $\$ 550$ saved towards her vacation and can earn $\$ 25$ per hour as an assistant in her uncle's photography studio. How many hours must she work in order to have enough money for her vacation?

### 2.6 Solve Compound Inequalities

Solve Compound Inequalities with "and"
In each of the following exercises, solve each inequality, graph the solution, and write the solution in interval notation.
593. $x \leq 5$ and $x>-3$
594. $4 x-2 \leq 4$ and
595. $5(3 x-2) \leq 5$ and $7 x-1>-8$
$4(x+2)<3$
596. $\frac{3}{4}(x-8) \leq 3$ and
597. $\frac{3}{4} x-5 \geq-2$ and
598. $-5 \leq 4 x-1<7$
$\frac{1}{5}(x-5) \leq 3$ $-3(x+1) \geq 6$

Solve Compound Inequalities with "or"
In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.
599. $5-2 x \leq-1$ or
$6+3 x \leq 4$
600. $3(2 x-3)<-5$ or
$4 x-1>3$
603. $\frac{1}{2} x-3 \leq 4$ or
602. $2(x+3) \geq 0$ or
$3(x+4) \leq 6$

$$
\frac{1}{3}(x-6) \geq-2
$$

601. $\frac{3}{4} x-2>4$ or $4(2-x)>0$

Solve Applications with Compound Inequalities
In the following exercises, solve.
604. Liam is playing a number game with his sister Audry. Liam is thinking of a number and wants Audry to guess it. Five more than three times her number is between 2 and 32. Write a compound inequality that shows the range of numbers that Liam might be thinking of.
605. Elouise is creating a rectangular garden in her back yard. The length of the garden is 12 feet. The perimeter of the garden must be at least 36 feet and no more than 48 feet. Use a compound inequality to find the range of values for the width of the garden.

### 2.7 Solve Absolute Value Inequalities

Solve Absolute Value Equations
In the following exercises, solve.
606. $|x|=8$
607. $|y|=-14$
608. $|z|=0$
609. $|3 x-4|+5=7$
610. $4|x-1|+2=10$
611. $-2|x-3|+8=-4$
612. $\left|\frac{1}{2} x+5\right|+4=1$
613. $|6 x-5|=|2 x+3|$

Solve Absolute Value Inequalities with "less than"
In the following exercises, solve each inequality. Graph the solution and write the solution in interval notation.
614. $|x| \leq 8$
615. $|2 x-5| \leq 3$
616. $|6 x-5|<7$
617. $|5 x+1| \leq-2$

Solve Absolute Value Inequalities with "greater than"
In the following exercises, solve. Graph the solution and write the solution in interval notation.
618. $|x|>6$
619. $|x| \geq 2$
620. $|x-5|>-2$
621. $|x-7| \geq 1$
622. $3|x|+4 \geq 1$

Solve Applications with Absolute Value
In the following exercises, solve.
623. A craft beer brewer needs 215,000 bottle per day. But this total can vary by as much as 5,000 bottles. What is the maximum and minimum expected usage at the bottling company?
624. At Fancy Grocery, the ideal weight of a loaf of bread is 16 ounces. By law, the actual weight can vary from the ideal by 1.5 ounces. What range of weight will be acceptable to the inspector without causing the bakery being fined?

## PRACTICE TEST

In the following exercises, solve each equation.
625. $-5(2 x+1)=45$
626.
$\frac{1}{4}(12 m+28)=6+2(3 m+1)$
627.
$8(3 a+5)-7(4 a-3)=20-3 a$
628. $0.1 d+0.25(d+8)=4.1$
629.
630.
$14 n-3(4 n+5)=-9+2(n-8)$
$3(3 u+2)+4[6-8(u-1)]=3(u-2)$
631. $\frac{3}{4} x-\frac{2}{3}=\frac{1}{2} x+\frac{5}{6}$
632. $|3 x-4|=8$
633. $|2 x-1|=|4 x+3|$
634. Solve the formula $x+2 y=5$ for $y$.

In the following exercises, graph the inequality on the number line and write in interval notation.
635. $x \geq-3.5$
636. $x<\frac{11}{4}$
637. $-2 \leq x<5$

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.
638. $8 k \geq 5 k-120$
639. $3 c-10(c-2)<5 c+16$
640. $\frac{3}{4} x-5 \geq-2$ and
$-3(x+1) \geq 6$
641. $3(2 x-3)<-5$ or
$4 x-1>3$
642. $\frac{1}{2} x-3 \leq 4$ or
$\frac{1}{3}(x-6) \geq-2$

In the following exercises, translate to an equation or inequality and solve.
644. Four less than twice $x$ is 16 .
645. Find the length of the missing side.

647. The sum of two consecutive odd integers is -112 . Find the numbers.
648. Marcus bought a television on sale for $\$ 626.50$ The original price of the television was $\$ 895$. Find (a) the amount of discount and (b) the discount rate.
651. The measure of one angle of a triangle is twice the measure of the smallest angle. The measure of the third angle is three times the measure of the smallest angle. Find the measures of all three angles.
646. One number is four more than twice another. Their sum is -47 . Find the numbers.
649. Bonita has $\$ 2.95$ in dimes and quarters in her pocket. If she has five more dimes than quarters, how many of each coin does she have?
652. The length of a rectangle is five feet more than four times the width. The perimeter is 60 feet. Find the dimensions of the rectangle.


$$
\begin{aligned}
& f(x)=|x| \\
& \text { Domain: }(-\infty, \infty) \\
& \text { Range: }[0, \infty)
\end{aligned}
$$

## REVIEW EXERCISES

### 3.1 Graph Linear Equations in Two Variables

Plot Points in a Rectangular Coordinate System
In the following exercises, plot each point in a rectangular coordinate system.
391.
(a) $(-1,-5)$
(b) $(-3,4)$
(c) $(2,-3)$
(d) $\left(1, \frac{5}{2}\right)$
392.
(a) $(-2,0)$
(b) $(0,-4)$
(c) $(0,5)$
(d) $(3,0)$

In the following exercises, determine which ordered pairs are solutions to the given equations.
393. $5 x+y=10$;
394. $y=6 x-2$;
(a) $(5,1)$
(a) $(1,4)$
(b) $(2,0)$
(b) $\left(\frac{1}{3}, 0\right)$
(c) $(4,-10)$
(c) $(6,-2)$

## Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.
395. $y=4 x-3$
396. $y=-3 x$
397. $y=\frac{1}{2} x+3$
398. $y=-\frac{4}{5} x-1$
399. $x-y=6$
400. $2 x+y=7$
401. $3 x-2 y=6$

Graph Vertical and Horizontal lines
In the following exercises, graph each equation.
402. $y=-2$
403. $x=3$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.
404. $y=-2 x$ and $y=-2$
405. $y=\frac{4}{3} x$ and $y=\frac{4}{3}$

Find $\boldsymbol{x}$ - and $\boldsymbol{y}$-Intercepts
In the following exercises, find the $x$ - and $y$-intercepts.
406.

407.


In the following exercises, find the intercepts of each equation.
408. $x-y=-1$
409. $x+2 y=6$
410. $2 x+3 y=12$
411. $y=\frac{3}{4} x-12$
412. $y=3 x$

Graph a Line Using the Intercepts
In the following exercises, graph using the intercepts.
413. $-x+3 y=3$
414. $x-y=4$
415. $2 x-y=5$
416. $2 x-4 y=8$
417. $y=4 x$

### 3.2 Slope of a Line

Find the Slope of a Line
In the following exercises, find the slope of each line shown.
418.

420.

419.

421.


In the following exercises, find the slope of each line.
422. $y=2$
423. $x=5$
424. $x=-3$
425. $y=-1$

Use the Slope Formula to find the Slope of a Line between Two Points
In the following exercises, use the slope formula to find the slope of the line between each pair of points.
426. $(-1,-1),(0,5)$
427. (3.5), $(4,-1)$
428. $(-5,-2),(3,2)$
429. $(2,1),(4,6)$

## Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.
430. $(2,-2) ; m=\frac{5}{2}$
431. $(-3,4) ; m=-\frac{1}{3}$
432. $x$-intercept $-4 ; m=3$
433. $y$-intercept 1 ; $m=-\frac{3}{4}$

Graph a Line Using Its Slope and Intercept
In the following exercises, identify the slope and $y$-intercept of each line.
434. $y=-4 x+9$
435. $y=\frac{5}{3} x-6$
436. $5 x+y=10$
437. $4 x-5 y=8$

In the following exercises, graph the line of each equation using its slope and $y$-intercept.
438. $y=2 x+3$
439. $y=-x-1$
440. $y=-\frac{2}{5} x+3$
441. $4 x-3 y=12$

In the following exercises, determine the most convenient method to graph each line.
442. $x=5$
443. $y=-3$
444. $2 x+y=5$
445. $x-y=2$
446. $y=\frac{2}{2} x+2$
447. $y=\frac{3}{4} x-1$

## Graph and Interpret Applications of Slope-Intercept

448. Katherine is a private chef. The equation $C=6.5 m+42$ models the relation between her weekly cost, $C$, in dollars and the number of meals, $m$, that she serves.
(a) Find Katherine's cost for a week when she serves no meals.
(b) Find the cost for a week when she serves 14 meals.
(C) Interpret the slope and $C$-intercept of the equation.
(d) Graph the equation.
449. Marjorie teaches piano. The equation $P=35 h-250$ models the relation between her weekly profit, $P$, in dollars and the number of student lessons, $s$, that she teaches.
(a) Find Marjorie's profit for a week when she teaches no student lessons.
(b) Find the profit for a week when she teaches 20 student lessons.
(c) Interpret the slope and $P$-intercept of the equation.
(d) Graph the equation.

Use Slopes to Identify Parallel and Perpendicular Lines
In the following exercises, use slopes and y-intercepts to determine if the lines are parallel, perpendicular, or neither.
450. $4 x-3 y=-1 ; y=\frac{4}{3} x-3$
451. $y=5 x-1 ; 10 x+2 y=0$
452. $3 x-2 y=5 ; 2 x+3 y=6$
453. $2 x-y=8 ; x-2 y=4$

### 3.3 Find the Equation of a Line

Find an Equation of the Line Given the Slope and $y$-Intercept
In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope-intercept form.
454. slope $\frac{1}{3}$ and $y$-intercept
455. slope -5 and $y$-intercept
456. slope 0 and $y$-intercept
(0, -6)
$(0,-3)$
$(0,4)$
457. slope -2 and $y$-intercept $(0,0)$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope-intercept form.
458.

459.

461.


Find an Equation of the Line Given the Slope and a Point
In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope-intercept form.
462. $m=-\frac{1}{4}$, point $(-8,3)$
463. $m=\frac{3}{5}$, point $(10,6)$
464. Horizontal line containing ( $-2,7$ )
465. $m=-2$, point $(-1,-3)$

Find an Equation of the Line Given Two Points
In the following exercises, find the equation of a line containing the given points. Write the equation in slope-intercept form.
466. $(2,10)$ and $(-2,-2)$
468. $(3,8)$ and $(3,-4)$
467. $(7,1)$ and $(5,0)$
469. $(5,2)$ and ( $-1,2$ )

Find an Equation of a Line Parallel to a Given Line
In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope-intercept form.
470. line $y=-3 x+6$, point $(1,-5)$
471. line $2 x+5 y=-10$, point $(10,4)$
472. line $x=4$, point $(-2,-1)$
473. line $y=-5$, point $(-4,3)$

## Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope-intercept form.
474. line $y=-\frac{4}{5} x+2$, point $(8,9)$
475. line $2 x-3 y=9$, point $(-4,0)$
476. line $y=3$, point $(-1,-3)$
477. line $x=-5$ point $(2,1)$

### 3.4 Graph Linear Inequalities in Two Variables

## Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.
478. Determine whether each ordered pair is a 479. Determine whether each ordered pair is a solution to the inequality $y<x-3$ : solution to the inequality $x+y>4$ :
(a) $(0,1)$
(b) $(-2,-4)$
(c) $(5,2)$
(d) $(3,-1)$
(a) $(6,1)$ (b) $(-3,6)$
(c) $(3,2)$
(d) $(-5,10)$
(e) $(0,0)$
(e) $(-1,-5)$

Recognize the Relation Between the Solutions of an Inequality and its Graph In the following exercises, write the inequality shown by the shaded region.
480. Write the inequality shown by the graph with the boundary line $y=-x+2$.

481. Write the inequality shown by the graph with the boundary line $y=\frac{2}{3} x-3$.

482. Write the inequality shown by the shaded region in the graph with the boundary line $x+y=-4$.

483. Write the inequality shown by the shaded region in the graph with the boundary line $x-2 y=6$.


Graph Linear Inequalities in Two Variables
In the following exercises, graph each linear inequality.
484. Graph the linear inequality $y>\frac{2}{5} x-4$.
485. Graph the linear inequality $y \leq-\frac{1}{4} x+3$.
486. Graph the linear inequality $x-y \leq 5$.
487. Graph the linear inequality $3 x+2 y>10$.
488. Graph the linear inequality $y \leq-3 x$.
489. Graph the linear inequality $y<6$.

Solve Applications using Linear Inequalities in Two Variables
490. Shanthie needs to earn at least $\$ 500$ a week during her summer break to pay for college. She works two jobs. One as a swimming instructor that pays $\$ 10$ an hour and the other as an intern in a law office for $\$ 25$ hour. How many hours does Shanthie need to work at each job to earn at least $\$ 500$ per week?
(a) Let $x$ be the number of hours she works teaching swimming and let $y$ be the number of hours she works as an intern. Write an inequality that would model this situation.
(b) Graph the inequality.
(c) Find three ordered pairs $(x, y)$ that would be solutions to the inequality. Then, explain what that means for Shanthie.
491. Atsushi he needs to exercise enough to burn 600 calories each day. He prefers to either run or bike and burns 20 calories per minute while running and 15 calories a minute while biking.
(a) If $x$ is the number of minutes that Atsushi runs and $y$ is the number minutes he bikes, find the inequality that models the situation.
(b) Graph the inequality.
(c) List three solutions to the inequality. What options do the solutions provide Atsushi?

### 3.5 Relations and Functions

## Find the Domain and Range of a Relation

In the following exercises, for each relation, (a) find the domain of the relation (b) find the range of the relation.
492. $\{(5,-2),(5,-4),(7,-6)$,
493. $\{(-3,7),(-2,3),(-1,9)$,
$(8,-8),(9,-10)\}$
$(0,-3),(-1,8)\}$

## PRACTICE TEST

537. Plot each point in a 538. Which of the given ordered rectangular coordinate system.
(a) $(2,5)$
(b) $(-1,-3)$
(c) $(0,2)$
(d) $\left(-4, \frac{3}{2}\right)$
(e) $(5,0)$

Find the slope of each line shown.
539. (a)

540. Find the slope of the line between the points $(5,2)$ and $(-1,-4)$.
(b)

542. Find the intercepts of $4 x+2 y=-8$ and graph.
pairs are solutions to the equation $3 x-y=6$ ?
(a) $(3,3)$ (b) $(2,0)$ © $(4,-6)$
541. Graph the line with slope $\frac{1}{2}$ containing the point $(-3,-4)$.

Graph the line for each of the following equations.
543. $y=\frac{5}{3} x-1$
544. $y=-x$
545. $y=2$

Find the equation of each line. Write the equation in slope-intercept form.
546. slope $-\frac{3}{4}$ and $y$-intercept $(0,-2)$
547. $m=2$, point $(-3,-1)$
548. containing $(10,1)$ and $(6,-1)$
549. perpendicular to the line $y=\frac{5}{4} x+2$, containing the point $(-10,3)$
550. Write the inequality shown by the graph with the boundary line $y=-x-3$.


Graph each linear inequality.
551. $y>\frac{3}{2} x+5$
553. $y \leq-5 x$
552. $x-y \geq-4$
554. Hiro works two part time jobs in order to earn enough money to meet her obligations of at least $\$ 450$ a week. Her job at the mall pays $\$ 10$ an hour and her administrative assistant job on campus pays $\$ 15$ an hour. How many hours does Hiro need to work at each job to earn at least $\$ 450$ ?
(a) Let $x$ be the number of hours she works at the mall and let $y$ be the number of hours she works as administrative assistant. Write an inequality that would model this situation.
(b) Graph the inequality .
(c) Find three ordered pairs $(x, y)$ that would be solutions to the inequality. Then explain what that means for Hiro.
555. Use the set of ordered pairs to (a) determine whether the relation is a function, (b) find the domain of the relation, and (c) find the range of the relation.
$\{(-3,27),(-2,8),(-1,1),(0,0)$,
$(1,1),(2,8),(3,27)\}$
556. Evaluate the function: (a) $f(-1)$ (b) $f(2)$ (c) $f(c)$.
$f(x)=4 x^{2}-2 x-3$

## REVIEW EXERCISES

### 4.1 Solve Systems of Linear Equations with Two Variables

Determine Whether an Ordered Pair is a Solution of a System of Equations.
In the following exercises, determine if the following points are solutions to the given system of equations.
328. $\left\{\begin{array}{l}x+3 y=-9 \\ 2 x-4 y=12\end{array}\right.$
329. $\left\{\begin{array}{l}x+y=8 \\ y=x-4\end{array}\right.$
(a) $(-3,-2)$
(a) $(6,2)$
(b) $(0,-3)$
(b) $(9,-1)$

Solve a System of Linear Equations by Graphing
In the following exercises, solve the following systems of equations by graphing.
330. $\left\{\begin{array}{l}3 x+y=6 \\ x+3 y=-6\end{array}\right.$
331. $\left\{\begin{array}{l}x+4 y=-1 \\ x=3\end{array}\right.$
332. $\left\{\begin{array}{l}2 x-y=5 \\ 4 x-2 y=10\end{array}\right.$
333. $\left\{\begin{array}{l}-x+2 y=4 \\ y=\frac{1}{2} x-3\end{array}\right.$

In the following exercises, without graphing determine the number of solutions and then classify the system of equations.
334. $\left\{\begin{array}{l}y=\frac{2}{5} x+2 \\ -2 x+5 y=10\end{array}\right.$
335. $\left\{\begin{array}{l}3 x+2 y=6 \\ y=-3 x+4\end{array}\right.$
336. $\left\{\begin{array}{l}5 x-4 y=0 \\ y=\frac{5}{4} x-5\end{array}\right.$

Solve a System of Equations by Substitution
In the following exercises, solve the systems of equations by substitution.
337. $\left\{\begin{array}{l}3 x-2 y=2 \\ y=\frac{1}{2} x+3\end{array}\right.$
338. $\left\{\begin{array}{l}x-y=0 \\ 2 x+5 y=-14\end{array}\right.$
339. $\left\{\begin{array}{l}y=-2 x+7 \\ y=\frac{2}{3} x-1\end{array}\right.$
340. $\left\{\begin{array}{l}y=-5 x \\ 5 x+y=6\end{array}\right.$
341. $\left\{\begin{array}{l}y=-\frac{1}{3} x+2 \\ x+3 y=6\end{array}\right.$

## Solve a System of Equations by Elimination

In the following exercises, solve the systems of equations by elimination
342. $\left\{\begin{array}{l}x+y=12 \\ x-y=-10\end{array}\right.$
343. $\left\{\begin{array}{l}3 x-8 y=20 \\ x+3 y=1\end{array}\right.$
344. $\left\{\begin{array}{l}9 x+4 y=2 \\ 5 x+3 y=5\end{array}\right.$
345. $\left\{\begin{array}{l}\frac{1}{3} x-\frac{1}{2} y=1 \\ \frac{3}{4} x-y=\frac{5}{2}\end{array}\right.$
346. $\left\{\begin{array}{l}-x+3 y=8 \\ 2 x-6 y=-20\end{array}\right.$

## Choose the Most Convenient Method to Solve a System of Linear Equations

In the following exercises, decide whether it would be more convenient to solve the system of equations by substitution or elimination.
347. $\left\{\begin{array}{l}6 x-5 y=27 \\ 3 x+10 y=-24\end{array}\right.$
348. $\left\{\begin{array}{l}y=3 x-9 \\ 4 x-5 y=23\end{array}\right.$

## PRACTICE TEST

In the following exercises, solve the following systems by graphing.
407. $\left\{\begin{array}{l}x-y=5 \\ x+2 y=-4\end{array}\right.$
408. $\left\{\begin{array}{l}x-y>-2 \\ y \leq 3 x+1\end{array}\right.$

In the following exercises, solve each system of equations. Use either substitution or elimination.
409. $\left\{\begin{array}{l}x+4 y=6 \\ -2 x+y=-3\end{array}\right.$
410. $\left\{\begin{array}{l}-3 x+4 y=25 \\ x-5 y=-23\end{array}\right.$
411. $\left\{\begin{array}{l}x+y-z=-1 \\ 2 x-y+2 z=8 \\ -3 x+2 y+z=-9\end{array}\right.$

Solve the system of equations using a matrix.
412. $\left\{\begin{array}{l}2 x+y=7 \\ x-2 y=6\end{array}\right.$
413. $\left\{\begin{array}{l}-3 x+y+z=-4 \\ -x+2 y-2 z=1 \\ 2 x-y-z=-1\end{array}\right.$

Solve using Cramer's rule.
414. $\left\{\begin{array}{l}3 x+y=-3 \\ 2 x+3 y=6\end{array}\right.$
415. Evaluate the determinant by expanding by minors:
$\left|\begin{array}{rrr}3 & -2 & -2 \\ 2 & -1 & 4 \\ -1 & 0 & -3\end{array}\right|$.

In the following exercises, translate to a system of equations and solve.
416. Greg is paddling his canoe upstream, against the current, to a fishing spot 10 miles away. If he paddles upstream for 2.5 hours and his return trip takes 1.25 hours, find the speed of the current and his paddling speed in still water.
419. The church youth group is selling snacks to raise money to attend their convention. Amy sold 2 pounds of candy, 3 boxes of cookies and 1 can of popcorn for a total sales of $\$ 65$. Brian sold 4 pounds of candy, 6 boxes of cookies and 3 cans of popcorn for a total sales of $\$ 140$. Paulina sold 8 pounds of candy, 8 boxes of cookies and 5 can of popcorn for a total sales of $\$ 250$. What is the cost of each item?
417. A pharmacist needs 20 liters of a $2 \%$ saline solution. He has a $1 \%$ and a $5 \%$ solution available. How many liters of the $1 \%$ and how many liters of the 5\% solutions should she mix to make the $2 \%$ solution?
420. The manufacturer of a granola bar spends $\$ 1.20$ to make each bar and sells them for $\$ 2$. The manufacturer also has fixed costs each month of $\$ 8,000$.
(a) Find the cost function $C$ when $x$ granola bars are manufactured
(b) Find the revenue function $R$ when $x$ granola bars are sold.
(c) Show the break-even point by graphing both the Revenue and Cost functions on the same grid.
(d) Find the break-even point. Interpret what the break-even point means.
418. Arnold invested $\$ 64,000$, some at $5.5 \%$ interest and the rest at $9 \%$. How much did he invest at each rate if he received $\$ 4,500$ in interest in one year?
421. Translate to a system of inequalities and solve.
Andi wants to spend no more than $\$ 50$ on Halloween treats. She wants to buy candy bars that cost \$1 each and lollipops that cost $\$ 0.50$ each, and she wants the number of lollipops to be at least three times the number of candy bars.
(a) Write a system of inequalities to model this situation.
(b) Graph the system.
(C) Can she buy 20 candy bars and 40 lollipops?

- Remainder Theorem
- If the polynomial function $f(x)$ is divided by $x-c$, then the remainder is $f(c)$.
- Factor Theorem: For any polynomial function $f(x)$,
- if $x-c$ is a factor of $f(x)$, then $f(c)=0$
- if $f(c)=0$, then $x-c$ is a factor of $f(x)$


## REVIEW EXERCISES

### 5.1 Add and Subtract Polynomials

Determine the Degree of Polynomials
In the following exercises, determine the type of polynomial.
342. $16 x^{2}-40 x-25$
344. -15

Add and Subtract Polynomials
In the following exercises, add or subtract the polynomials.
346. $4 p+11 p$
348. $\left(4 a^{2}+9 a-11\right)+\left(6 a^{2}-5 a+10\right)$
350. $\left(y^{2}-3 y+12\right)+\left(5 y^{2}-9\right)$

In the following exercises, simplify.
354. $17 m n^{2}-\left(-9 m n^{2}\right)+3 m n^{2}$
356. $2 p q^{2}-5 p-3 q^{2}$
358. $\left(3 p^{2}-4 p-9\right)+\left(5 p^{2}+14\right)$
360. $\left(7 b^{2}-4 b+3\right)-\left(8 b^{2}-5 b-7\right)$
362. Find the difference of $\left(z^{2}-4 z-12\right)$ and $\left(3 z^{2}+2 z-11\right)$
364. $\left(x^{3}-2 x^{2} y\right)-\left(x y^{2}-3 y^{3}\right)-\left(x^{2} y-4 x y^{2}\right)$
343. $5 m+9$
345. $y^{2}+6 y^{3}+9 y^{4}$
352. Find the sum of $8 q^{3}-27$ and $q^{2}+6 q-2$.
353. Find the difference of $x^{2}+6 x+8$ and $x^{2}-8 x+15$.
355. $18 a-7 b-21 a$
357. $\left(6 a^{2}+7\right)+\left(2 a^{2}-5 a-9\right)$
359. $\left(7 m^{2}-2 m-5\right)-\left(4 m^{2}+m-8\right)$
361. Subtract $\left(8 y^{2}-y+9\right)$ from $\left(11 y^{2}-9 y-5\right)$
363. $\left(x^{3}-x^{2} y\right)-\left(4 x y^{2}-y^{3}\right)+\left(3 x^{2} y-x y^{2}\right)$

Evaluate a Polynomial Function for a Given Value of the Variable
In the following exercises, find the function values for each polynomial function.
365. For the function $f(x)=7 x^{2}-3 x+5$ find:
(a) $f(5)$ (b) $f(-2)$ © $f(0)$
367. A pair of glasses is dropped off a bridge 640 feet above a river. The polynomial function $h(t)=-16 t^{2}+640$ gives the height of the glasses $t$ seconds after they were dropped. Find the height of the glasses when $t=6$.
366. For the function $g(x)=15-16 x^{2}$, find:
(a) $g(-1)$ (b) $g(0)$ © $g(2)$
368. A manufacturer of the latest soccer shoes has found that the revenue received from selling the shoes at a cost of $p$ dollars each is given by the polynomial $R(p)=-5 p^{2}+360 p$. Find the revenue received when $p=110$ dollars.

## Add and Subtract Polynomial Functions

In the following exercises, find © $(f+g)(x)$ (b) $(f+g)(3)$ © $(f-g)(x)$ (d) $(f-g)(-2)$
369. $f(x)=2 x^{2}-4 x-7$ and $g(x)=2 x^{2}-x+5 \quad$ 370. $f(x)=4 x^{3}-3 x^{2}+x-1$ and $g(x)=8 x^{3}-1$

### 5.2 Properties of Exponents and Scientific Notation

## Simplify Expressions Using the Properties for Exponents

In the following exercises, simplify each expression using the properties for exponents.
371. $p^{3} \cdot p^{10}$
372. $2 \cdot 2^{6}$
373. $a \cdot a^{2} \cdot a^{3}$
374. $x \cdot x^{8}$
375. $y^{a} \cdot y^{b}$
376. $\frac{2^{8}}{2^{2}}$
377. $\frac{a^{6}}{a}$
378. $\frac{n^{3}}{n^{12}}$
379. $\frac{1}{x^{5}}$
380. $3^{0}$
381. $y^{0}$
382. $(14 t)^{0}$
383. $12 a^{0}-15 b^{0}$

Use the Definition of a Negative Exponent
In the following exercises, simplify each expression.
384. $6^{-2}$
385. $(-10)^{-3}$
386. $5 \cdot 2^{-4}$
387. $(8 n)^{-1}$
388. $y^{-5}$
389. $10^{-3}$
390. $\frac{1}{a^{-4}}$
391. $\frac{1}{6^{-2}}$
392. $-5^{-3}$
393. $\left(-\frac{1}{5}\right)^{-3}$
394. $-\left(\frac{1}{2}\right)^{-3}$
395. $(-5)^{-3}$
396. $\left(\frac{5}{9}\right)^{-2}$
397. $\left(-\frac{3}{x}\right)^{-3}$

In the following exercises, simplify each expression using the Product Property.
398. $\left(y^{4}\right)^{3}$
399. $\left(3^{2}\right)^{5}$
400. $\left(a^{10}\right)^{y}$
401. $x^{-3} \cdot x^{9}$
402. $r^{-5} \cdot r^{-4}$
403. $\left(u v^{-3}\right)\left(u^{-4} v^{-2}\right)$
404. $\left(m^{5}\right)^{-1}$
405. $p^{5} \cdot p^{-2} \cdot p^{-4}$

In the following exercises, simplify each expression using the Power Property.
406. $\left(k^{-2}\right)^{-3}$
407. $\frac{q^{4}}{q^{20}}$
408. $\frac{b^{8}}{b^{-2}}$
409. $\frac{n^{-3}}{n^{-5}}$

In the following exercises, simplify each expression using the Product to a Power Property.
410. $(-5 a b)^{3}$
411. $(-4 p q)^{0}$
412. $\left(-6 x^{3}\right)^{-2}$
413. $\left(3 y^{-4}\right)^{2}$

In the following exercises, simplify each expression using the Quotient to a Power Property.
414. $\left(\frac{3}{5 x}\right)^{-2}$
415. $\left(\frac{3 x y^{2}}{z}\right)^{4}$
416. $\left(\frac{4 p^{-3}}{q^{2}}\right)^{2}$

In the following exercises, simplify each expression by applying several properties.
417. $\left(x^{2} y\right)^{2}\left(3 x y^{5}\right)^{3}$
418. $\frac{\left(-3 a^{-2}\right)^{4}\left(2 a^{4}\right)^{2}}{\left(-6 a^{2}\right)^{3}}$
419. $\left(\frac{3 x y^{3}}{4 x^{4} y^{-2}}\right)^{2}\left(\frac{6 x y^{4}}{8 x^{3} y^{-2}}\right)^{-1}$

In the following exercises, write each number in scientific notation.
420. 2.568
421. 5,300,000
422. 0.00814

In the following exercises, convert each number to decimal form.
423. $2.9 \times 10^{4}$
424. $3.75 \times 10^{-1}$
425. $9.413 \times 10^{-5}$

In the following exercises, multiply or divide as indicated. Write your answer in decimal form.
426. $\left(3 \times 10^{7}\right)\left(2 \times 10^{-4}\right)$
427. $\left(1.5 \times 10^{-3}\right)\left(4.8 \times 10^{-1}\right)$
428. $\frac{6 \times 10^{9}}{2 \times 10^{-1}}$
429. $\frac{9 \times 10^{-3}}{1 \times 10^{-6}}$

### 5.3 Multiply Polynomials

## Multiply Monomials

In the following exercises, multiply the monomials.
430. $\left(-6 p^{4}\right)(9 p)$
431. $\left(\frac{1}{3} c^{2}\right)\left(30 c^{8}\right)$
432. $\left(8 x^{2} y^{5}\right)\left(7 x y^{6}\right)$
433. $\left(\frac{2}{3} m^{3} n^{6}\right)\left(\frac{1}{6} m^{4} n^{4}\right)$

Multiply a Polynomial by a Monomial
In the following exercises, multiply.
434. $7(10-x)$
435. $a^{2}\left(a^{2}-9 a-36\right)$
436. $-5 y\left(125 y^{3}-1\right)$
437. $(4 n-5)\left(2 n^{3}\right)$

Multiply a Binomial by a Binomial
In the following exercises, multiply the binomials using:
(a) the Distributive Property (b) the FOIL method (c) the Vertical Method.
438. $(a+5)(a+2)$
439. $(y-4)(y+12)$
440. $(3 x+1)(2 x-7)$
441. $(6 p-11)(3 p-10)$

In the following exercises, multiply the binomials. Use any method.
442. $(n+8)(n+1)$
443. $(k+6)(k-9)$
444. $(5 u-3)(u+8)$
445. $(2 y-9)(5 y-7)$
446. $(p+4)(p+7)$
447. $(x-8)(x+9)$
448. $(3 c+1)(9 c-4)$
449. $(10 a-1)(3 a-3)$

Multiply a Polynomial by a Polynomial
In the following exercises, multiply using (a) the Distributive Property (b) the Vertical Method.
450. $(x+1)\left(x^{2}-3 x-21\right)$
451. $(5 b-2)\left(3 b^{2}+b-9\right)$

In the following exercises, multiply. Use either method.
452. $(m+6)\left(m^{2}-7 m-30\right)$
453. $(4 y-1)\left(6 y^{2}-12 y+5\right)$

## Multiply Special Products

In the following exercises, square each binomial using the Binomial Squares Pattern.
454. $(2 x-y)^{2}$
455. $\left(x+\frac{3}{4}\right)^{2}$
456. $\left(8 p^{3}-3\right)^{2}$
457. $(5 p+7 q)^{2}$

In the following exercises, multiply each pair of conjugates using the Product of Conjugates.
458. $(3 y+5)(3 y-5)$
459. $(6 x+y)(6 x-y)$
460. $\left(a+\frac{2}{3} b\right)\left(a-\frac{2}{3} b\right)$
461. $\left(12 x^{3}-7 y^{2}\right)\left(12 x^{3}+7 y^{2}\right)$
462. $\left(13 a^{2}-8 b^{4}\right)\left(13 a^{2}+8 b^{4}\right)$

### 5.4 Divide Monomials

## Divide Monomials

In the following exercises, divide the monomials.
463. $72 p^{12} \div 8 p^{3}$
464. $-26 a^{8} \div\left(2 a^{2}\right)$
465. $\frac{45 y^{6}}{-15 y^{10}}$
466. $\frac{-30 x^{8}}{-36 x^{9}}$
467. $\frac{28 a^{9} b}{7 a^{4} b^{3}}$
468. $\frac{11 u^{6} v^{3}}{55 u^{2} v^{8}}$
469. $\frac{\left(5 m^{9} n^{3}\right)\left(8 m^{3} n^{2}\right)}{\left(10 m n^{4}\right)\left(m^{2} n^{5}\right)}$
470. $\frac{\left(42 r^{2} s^{4}\right)\left(54 r s^{2}\right)}{\left(6 r s^{3}\right)(9 s)}$

## Divide a Polynomial by a Monomial

In the following exercises, divide each polynomial by the monomial
471. $\left(54 y^{4}-24 y^{3}\right) \div\left(-6 y^{2}\right)$
472.
$\frac{63 x^{3} y^{2}-99 x^{2} y^{3}-45 x^{4} y^{3}}{9 x^{2} y^{2}}$
473. $\frac{12 x^{2}+4 x-3}{-4 x}$

Divide Polynomials using Long Division
In the following exercises, divide each polynomial by the binomial.
474. $\left(4 x^{2}-21 x-18\right) \div(x-6)$
475. $\left(y^{2}+2 y+18\right) \div(y+5)$
476.
$\left(n^{3}-2 n^{2}-6 n+27\right) \div(n+3)$
477. $\left(a^{3}-1\right) \div(a+1)$

Divide Polynomials using Synthetic Division
In the following exercises, use synthetic Division to find the quotient and remainder.
478. $x^{3}-3 x^{2}-4 x+12$
is 479. $2 x^{3}-11 x^{2}+11 x+12$ is
480. $x^{4}+x^{2}+6 x-10 \quad$ is divided by $x-3$
divided by $x+2$

## Divide Polynomial Functions

In the following exercises, divide.
481. For functions $f(x)=x^{2}-15 x+45$ and
482. For functions $f(x)=x^{3}+x^{2}-7 x+2$ and $g(x)=x-9$, find (a) $\left(\frac{f}{g}\right)(x)$ $g(x)=x-2$, find (a) $\left(\frac{f}{g}\right)(x)$
(b) $\left(\frac{f}{g}\right)(-2)$
(b) $\left(\frac{f}{g}\right)(3)$

## PRACTICE TEST

487. For the polynomial
$8 y^{4}-3 y^{2}+1$$\quad \begin{aligned} & 488 . \\ & \left(5 a^{2}+2 a-12\right)\left(9 a^{2}+8 a-4\right)\end{aligned}$
(a) Is it a monomial, binomial, or trinomial? (b) What is its degree?
488. $\left(-\frac{3}{4}\right)^{3}$
489. $x^{-3} x^{4}$
490. $\left(47 a^{18} b^{23} c^{5}\right)^{0}$
491. $p^{-3} \cdot p^{-8}$
492. $\frac{x^{4}}{x^{-5}}$
493. $\frac{24 r^{3} s}{6 r^{2} s^{7}}$
494. $4 u\left(u^{2}-9 u+1\right)$
495. $(4 x-3)^{2}$
496. 

$\left(3 x^{3}-10 x^{2}+7 x+10\right) \div(3 x+2)$
500. $\left(\frac{x^{4} y^{9}}{x^{-3}}\right)^{2}$
503. $(m+3)(7 m-2)$
506. $(5 x+2 y)(5 x-2 y)$
509. Use the Factor Theorem to determine if $x+3$ a factor of $x^{3}+8 x^{2}+21 x+18$.

In the following exercises, simplify and write your answer in decimal form.
511. $\left(2.4 \times 10^{8}\right)\left(2 \times 10^{-5}\right)$
512. $\frac{9 \times 10^{4}}{3 \times 10^{-1}}$
515. For functions
514. For $f(x)=2 x^{2}-3 x-5$ and $g(x)=3 x^{2}-4 x+1$, find
$f(x)=3 x^{2}-23 x-36$ and $g(x)=x-9$, find
(a) $(f+g)(x)$ (b) $(f+g)(1)$
(c) $(f-g)(x)$ (d) $(f-g)(-2)$
(a) $\left(\frac{f}{g}\right)(x)$ (b) $\left(\frac{f}{g}\right)(3)$
489. $\left(10 x^{2}-3 x+5\right)-\left(4 x^{2}-6\right)$
492. $\frac{5^{6}}{5^{8}}$
495. $(2 y)^{-3}$
498. $\left(3 x^{-3}\right)^{2}$
501. $\left(8 x y^{3}\right)\left(-6 x^{4} y^{6}\right)$
504. $(n-8)\left(n^{2}-4 n+11\right)$
507. $\left(15 x y^{3}-35 x^{2} y\right) \div 5 x y$
510. (a) Convert 112,000 to scientific notation. (b) Convert $5.25 \times 10^{-4}$ to decimal form.
513. For the function $f(x)=6 x^{2}-3 x-9$ find:
(a) $f(3)$ (b) $f(-2)$ © $f(0)$
516. A hiker drops a pebble from a bridge 240 feet above a canyon. The function $h(t)=-16 t^{2}+240$ gives the height of the pebble $t$ seconds after it was dropped. Find the height when $t=3$.
$\square$

### 6.2 EXERCISES

## Practice Makes Perfect

Factor Trinomials of the Form $x^{2}+b x+c$
In the following exercises, factor each trinomial of the form $x^{2}+b x+c$.
61. $p^{2}+11 p+30$
62. $w^{2}+10 x+21$
63. $n^{2}+19 n+48$
64. $b^{2}+14 b+48$
65. $a^{2}+25 a+100$
66. $u^{2}+101 u+100$
67. $x^{2}-8 x+12$
68. $q^{2}-13 q+36$
69. $y^{2}-18 x+45$
70. $m^{2}-13 m+30$
71. $x^{2}-8 x+7$
72. $y^{2}-5 y+6$
73. $5 p-6+p^{2}$
74. $6 n-7+n^{2}$
75. $8-6 x+x^{2}$
76. $7 x+x^{2}+6$
77. $x^{2}-12-11 x$
78. $-11-10 x+x^{2}$

In the following exercises, factor each trinomial of the form $x^{2}+b x y+c y^{2}$.
79. $x^{2}-2 x y-80 y^{2}$
80. $p^{2}-8 p q-65 q^{2}$
81. $m^{2}-64 m n-65 n^{2}$
82. $p^{2}-2 p q-35 q^{2}$
83. $a^{2}+5 a b-24 b^{2}$
84. $r^{2}+3 r s-28 s^{2}$
85. $x^{2}-3 x y-14 y^{2}$
86. $u^{2}-8 u v-24 v^{2}$
87. $m^{2}-5 m n+30 n^{2}$
88. $c^{2}-7 c d+18 d^{2}$

Factor Trinomials of the Form $a x^{2}+b x+c$ Using Trial and Error In the following exercises, factor completely using trial and error.
89. $p^{3}-8 p^{2}-20 p$
90. $q^{3}-5 q^{2}-24 q$
91. $3 m^{3}-21 m^{2}+30 m$
92. $11 n^{3}-55 n^{2}+44 n$
93. $5 x^{4}+10 x^{3}-75 x^{2}$
94. $6 y^{4}+12 y^{3}-48 y^{2}$
95. $2 t^{2}+7 t+5$
96. $5 y^{2}+16 y+11$
97. $11 x^{2}+34 x+3$
98. $7 b^{2}+50 b+7$
99. $4 w^{2}-5 w+1$
100. $5 x^{2}-17 x+6$
101. $4 q^{2}-7 q-2$
102. $10 y^{2}-53 y-11$
103. $6 p^{2}-19 p q+10 q^{2}$
104. $21 m^{2}-29 m n+10 n^{2}$
105. $4 a^{2}+17 a b-15 b^{2}$
106. $6 u^{2}+5 u v-14 v^{2}$
107. $-16 x^{2}-32 x-16$
108. $-81 a^{2}+153 a+18$
109. $-30 q^{3}-140 q^{2}-80 q$
110. $-5 y^{3}-30 y^{2}+35 y$

Factor Trinomials of the Form $a x^{2}+b x+c$ using the 'ac' Method In the following exercises, factor using the 'ac' method.
111. $5 n^{2}+21 n+4$
112. $8 w^{2}+25 w+3$
113. $4 k^{2}-16 k+15$
114. $5 s^{2}-9 s+4$
115. $6 y^{2}+y-15$
116. $6 p^{2}+p-22$
117. $2 n^{2}-27 n-45$
118. $12 z^{2}-41 z-11$
119. $60 y^{2}+290 y-50$
120. $6 u^{2}-46 u-16$
121. $48 z^{3}-102 z^{2}-45 z$
122. $90 n^{3}+42 n^{2}-216 n$
123. $16 s^{2}+40 s+24$
124. $24 p^{2}+160 p+96$
126. $30 x^{2}+105 x-60$

## Factor Using Substitution

In the following exercises, factor using substitution.
127. $x^{4}-x^{2}-12$
128. $x^{4}+2 x^{2}-8$
129. $x^{4}-3 x^{2}-28$
130. $x^{4}-13 x^{2}-30$
131. $(x-3)^{2}-5(x-3)-36$
132. $(x-2)^{2}-3(x-2)-54$
133. $(3 y-2)^{2}-(3 y-2)-2$
134. $(5 y-1)^{2}-3(5 y-1)-18$

Mixed Practice
In the following exercises, factor each expression using any method.
135. $u^{2}-12 u+36$
136. $x^{2}-14 x-32$
137. $r^{2}-20 r s+64 s^{2}$
138. $q^{2}-29 q r-96 r^{2}$
139. $12 y^{2}-29 y+14$
140. $12 x^{2}+36 y-24 z$
141. $6 n^{2}+5 n-4$
142. $3 q^{2}+6 q+2$
143. $13 z^{2}+39 z-26$
144. $5 r^{2}+25 r+30$
145. $3 p^{2}+21 p$
146. $7 x^{2}-21 x$
147. $6 r^{2}+30 r+36$
148. $18 m^{2}+15 m+3$
149. $24 n^{2}+20 n+4$
150. $4 a^{2}+5 a+2$
151. $x^{4}-4 x^{2}-12$
152. $x^{4}-7 x^{2}-8$
153. $(x+3)^{2}-9(x+3)-36$
154. $(x+2)^{2}-25(x+2)-54$

### 6.5 EXERCISES

## Practice Makes Perfect

## Use the Zero Product Property

In the following exercises, solve.
277. $(3 a-10)(2 a-7)=0$
278. $(5 b+1)(6 b+1)=0$
279. $6 m(12 m-5)=0$
280. $2 x(6 x-3)=0$
281. $(2 x-1)^{2}=0$
282. $(3 y+5)^{2}=0$

Solve Quadratic Equations by Factoring
In the following exercises, solve.
283. $5 a^{2}-26 a=24$
284. $4 b^{2}+7 b=-3$
285. $4 m^{2}=17 m-15$
286. $n^{2}=5-6 n$
287. $7 a^{2}+14 a=7 a$
288. $12 b^{2}-15 b=-9 b$
289. $49 m^{2}=144$
290. $625=x^{2}$
291. $16 y^{2}=81$
292. $64 p^{2}=225$
293. $121 n^{2}=36$
294. $100 y^{2}=9$
295. $(x+6)(x-3)=-8$
296. $(p-5)(p+3)=-7$
297. $(2 x+1)(x-3)=-4 x$
298. $(y-3)(y+2)=4 y$
299. $(3 x-2)(x+4)=12 x$
300. $(2 y-3)(3 y-1)=8 y$
301. $20 x^{2}-60 x=-45$
302. $3 y^{2}-18 y=-27$
303. $15 x^{2}-10 x=40$
304. $14 y^{2}-77 y=-35$
305. $18 x^{2}-9=-21 x$
306. $16 y^{2}+12=-32 x$
307. $16 p^{3}=24 p^{2}+9 p$
308. $m^{3}-2 m^{2}=-m$
309. $2 x^{3}+72 x=24 x^{2}$
310. $3 y^{3}+48 y=24 y^{2}$
311. $36 x^{3}+24 x^{2}=-4 x$
312. $2 y^{3}+2 y^{2}=12 y$

Solve Equations with Polynomial Functions
In the following exercises, solve.
313. For the function, $f(x)=x^{2}-8 x+8$, (a) find 314. For the function, $f(x)=x^{2}+11 x+20$, (a) find
when $f(x)=-4$ (b) Use this information to find two points that lie on the graph of the function.
315. For the function, $f(x)=8 x^{2}-18 x+5$, (a) find when $f(x)=-4$ (b) Use this information to find two points that lie on the graph of the function.
when $f(x)=-8$ (b) Use this information to find two points that lie on the graph of the function.
316. For the function, $f(x)=18 x^{2}+15 x-10$, (a) find when $f(x)=15$ (b) Use this information to find two points that lie on the graph of the function.

In the following exercises, for each function, find: (a) the zeros of the function (b) the x-intercepts of the graph of the function (c) the $y$-intercept of the graph of the function.
317. $f(x)=9 x^{2}-4$
318. $f(x)=25 x^{2}-49$
319. $f(x)=6 x^{2}-7 x-5$

## Solve Applications Modeled by Quadratic Equations

In the following exercises, solve.
321. The product of two consecutive odd integers is 143. Find the integers.
323. The product of two consecutive even integers is 168. Find the integers.
325. The area of a rectangular carpet is 28 square feet. The length is three feet more than the width. Find the length and the width of the carpet.
327. The area of a bulletin board is 55 feet. The length is four feet less than three times the width. Find the length and the width of the a bulletin board.
329. A pennant is shaped like a right triangle, with hypotenuse 10 feet. The length of one side of the pennant is two feet longer than the length of the other side. Find the length of the two sides of the pennant.
331. A reflecting pool is shaped like a right triangle, with one leg along the wall of a building. The hypotenuse is 9 feet longer than the side along the building. The third side is 7 feet longer than the side along the building. Find the lengths of all three sides of the reflecting pool.
333. Juli is going to launch a model rocket in her back yard. When she launches the rocket, the function $h(t)=-16 t^{2}+32 t$ models the height, $h$, of the rocket above the ground as a function of time, $t$. Find:
(a) the zeros of this function which tells us when the penny will hit the ground. (b) the time the rocket will be 16 feet above the ground.

## Writing Exercises

335. Explain how you solve a quadratic equation. How many answers do you expect to get for a quadratic equation?
336. $f(x)=12 x^{2}-11 x+2$
337. The product of two consecutive odd integers is 195. Find the integers.
338. The product of two consecutive even integers is 288. Find the integers.
339. A rectangular retaining wall has area 15 square feet. The height of the wall is two feet less than its length. Find the height and the length of the wall.
340. A rectangular carport has area 150 square feet. The height of the carport is five feet less than twice its length. Find the height and the length of the carport.
341. A stained glass window is shaped like a right triangle. The hypotenuse is 15 feet. One leg is three more than the other. Find the lengths of the legs.
342. A goat enclosure is in the shape of a right triangle. One leg of the enclosure is built against the side of the barn. The other leg is 4 feet more than the leg against the barn. The hypotenuse is 8 feet more than the leg along the barn. Find the three sides of the goat enclosure.
343. Gianna is going to throw a ball from the top floor of her middle school. When she throws the ball from 48 feet above the ground, the function $h(t)=-16 t^{2}+32 t+48$ models the height, $h$, of the ball above the ground as a function of time, $t$. Find:
(a) the zeros of this function which tells us when the ball will hit the ground. (b) the time(s) the ball will be 48 feet above the ground. (c) the height the ball will be at $t=1$ seconds which is when the ball will be at its highest point.
344. Give an example of a quadratic equation that has a GCF and none of the solutions to the equation is zero.

Step 4. Solve the linear equations.
Step 5. Check. Substitute each solution separately into the original equation.

- Zero of a Function: For any function $f$, if $f(x)=0$, then $x$ is a zero of the function.
- How to use a problem solving strategy to solve word problems.

Step 1. Read the problem. Make sure all the words and ideas are understood.
Step 2. Identify what we are looking for.
Step 3. Name what we are looking for. Choose a variable to represent that quantity.
Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.

Step 5. Solve the equation using appropriate algebra techniques.
Step 6. Check the answer in the problem and make sure it makes sense.
Step 7. Answer the question with a complete sentence.

## REVIEW EXERCISES

### 6.1 Greatest Common Factor and Factor by Grouping

Find the Greatest Common Factor of Two or More Expressions
In the following exercises, find the greatest common factor.
337. $12 a^{2} b^{3}, 15 a b^{2}$
338. $12 m^{2} n^{3}, 42 m^{5} n^{3}$
339. $15 y^{3}, 21 y^{2}, 30 y$
340. $45 x^{3} y^{2}, 15 x^{4} y, 10 x^{5} y^{3}$

Factor the Greatest Common Factor from a Polynomial
In the following exercises, factor the greatest common factor from each polynomial.
341. $35 y+84$
342. $6 y^{2}+12 y-6$
343. $18 x^{3}-15 x$
344. $15 m^{4}+6 m^{2} n$
345. $4 x^{3}-12 x^{2}+16 x$
346. $-3 x+24$
347. $-3 x^{3}+27 x^{2}-12 x$
348. $3 x(x-1)+5(x-1)$

Factor by Grouping
In the following exercises, factor by grouping.
349. $a x-a y+b x-b y$
350. $x^{2} y-x y^{2}+2 x-2 y$
351. $x^{2}+7 x-3 x-21$
352. $4 x^{2}-16 x+3 x-12$
353. $m^{3}+m^{2}+m+1$
354. $5 x-5 y-y+x$

### 6.2 Factor Trinomials

Factor Trinomials of the Form $x^{2}+b x+c$
In the following exercises, factor each trinomial of the form $x^{2}+b x+c$.
355. $a^{2}+14 a+33$
356. $k^{2}-16 k+60$
357. $m^{2}+3 m-54$
358. $x^{2}-3 x-10$

In the following examples, factor each trinomial of the form $x^{2}+b x y+c y^{2}$.
359. $x^{2}+12 x y+35 y^{2}$
360. $r^{2}+3 r s-28 s^{2}$
361. $a^{2}+4 a b-21 b^{2}$
362. $p^{2}-5 p q-36 q^{2}$
363. $m^{2}-5 m n+30 n^{2}$

Factor Trinomials of the Form $a x^{2}+b x+c$ Using Trial and Error In the following exercises, factor completely using trial and error.
364. $x^{3}+5 x^{2}-24 x$
365. $3 y^{3}-21 y^{2}+30 y$
366. $5 x^{4}+10 x^{3}-75 x^{2}$
367. $5 y^{2}+14 y+9$
368. $8 x^{2}+25 x+3$
369. $10 y^{2}-53 y-11$
370. $6 p^{2}-19 p q+10 q^{2}$
371. $-81 a^{2}+153 a+18$

Factor Trinomials of the Form $a x^{2}+b x+c$ using the ' $a c^{\prime}$ Method In the following exercises, factor.
372. $2 x^{2}+9 x+4$
373. $18 a^{2}-9 a+1$
374. $15 p^{2}+2 p-8$
375. $15 x^{2}+6 x-2$
376. $8 a^{2}+32 a+24$
377. $3 x^{2}+3 x-36$
378. $48 y^{2}+12 y-36$
379. $18 a^{2}-57 a-21$
380. $3 n^{4}-12 n^{3}-96 n^{2}$

Factor using substitution
In the following exercises, factor using substitution.
381. $x^{4}-13 x^{2}-30$
382. $(x-3)^{2}-5(x-3)-36$

### 6.3 Factor Special Products

## Factor Perfect Square Trinomials

In the following exercises, factor completely using the perfect square trinomials pattern.
383. $25 x^{2}+30 x+9$
384. $36 a^{2}-84 a b+49 b^{2}$
385. $40 x^{2}+360 x+810$
386. $5 k^{3}-70 k^{2}+245 k$
387. $75 u^{4}-30 u^{3} v+3 u^{2} v^{2}$

## Factor Differences of Squares

In the following exercises, factor completely using the difference of squares pattern, if possible.
388. $81 r^{2}-25$
389. $169 m^{2}-n^{2}$
390. $25 p^{2}-1$
391. $9-121 y^{2}$
392. $20 x^{2}-125$
393. $169 n^{3}-n$
394. $6 p^{2} q^{2}-54 p^{2}$
395. $24 p^{2}+54$
396. $49 x^{2}-81 y^{2}$
397. $16 z^{4}-1$
398. $48 m^{4} n^{2}-243 n^{2}$
399. $a^{2}+6 a+9-9 b^{2}$
400. $x^{2}-16 x+64-y^{2}$

Factor Sums and Differences of Cubes
In the following exercises, factor completely using the sums and differences of cubes pattern, if possible.
401. $a^{3}-125$
402. $b^{3}-216$
403. $2 m^{3}+54$
404. $81 m^{3}+3$

### 6.4 General Strategy for Factoring Polynomials

Recognize and Use the Appropriate Method to Factor a Polynomial Completely In the following exercises, factor completely.
405. $24 x^{3}+44 x^{2}$
406. $24 a^{4}-9 a^{3}$
407. $16 n^{2}-56 m n+49 m^{2}$
408. $6 a^{2}-25 a-9$
409. $5 u^{4}-45 u^{2}$
410. $n^{4}-81$
411. $64 j^{2}+225$
412. $5 x^{2}+5 x-60$
413. $b^{3}-64$
414. $m^{3}+125$
415. $2 b^{2}-2 b c+5 c b-5 c^{2}$
416. $48 x^{5} y^{2}-243 x y^{2}$
417. $5 q^{2}-15 q-90$
418. $4 u^{5} v+4 u^{2} v^{3}$
419. $10 m^{4}-6250$
420. $60 x^{2} y-75 x y+30 y$
421. $16 x^{2}-24 x y+9 y^{2}-64$

### 6.5 Polynomial Equations

Use the Zero Product Property In the following exercises, solve.
422. $(a-3)(a+7)=0$
423. $(5 b+1)(6 b+1)=0$
424. $6 m(12 m-5)=0$
425. $(2 x-1)^{2}=0$
426. $3 m(2 m-5)(m+6)=0$

Solve Quadratic Equations by Factoring
In the following exercises, solve.
427. $x^{2}+9 x+20=0$
428. $y^{2}-y-72=0$
429. $2 p^{2}-11 p=40$
430. $q^{3}+3 q^{2}+2 q=0$
431. $144 m^{2}-25=0$
432. $4 n^{2}=36$
433. $(x+6)(x-3)=-8$
434. $(3 x-2)(x+4)=12 x$
435. $16 p^{3}=24 p^{2}+9 p$
436. $2 y^{3}+2 y^{2}=12 y$

Solve Equations with Polynomial Functions
In the following exercises, solve.
437. For the function, $f(x)=x^{2}+11 x+20$, (a) find when $f(x)=-8$ (b) Use this information to find two points that lie on the graph of the function.
438. For the function, $f(x)=9 x^{2}-18 x+5$, (a) find when $f(x)=-3$ (b) Use this information to find two points that lie on the graph of the function.

In each function, find: (a) the zeros of the function (b) the x-intercepts of the graph of the function (c) the y-intercept of the graph Download for free at http://cnx.org/contents/02776133-d49d-49cb-bfaa-67c7f61b25a1@6.1.
of the function.
439. $f(x)=64 x^{2}-49$

Solve Applications Modeled by Quadratic Equations
In the following exercises, solve.
441. The product of two consecutive numbers is 399. Find the numbers.
443. A ladder leans against the wall of a building. The length of the ladder is 9 feet longer than the distance of the bottom of the ladder from the building. The distance of the top of the ladder reaches up the side of the building is 7 feet longer than the distance of the bottom of the ladder from the building. Find the lengths of all three sides of the triangle formed by the ladder leaning against the building.
440. $f(x)=6 x^{2}-13 x-5$
442. The area of a rectangular shaped patio 432 square feet. The length of the patio is 6 feet more than its width. Find the length and width.
444. Shruti is going to throw a ball from the top of a cliff. When she throws the ball from 80 feet above the ground, the function $h(t)=-16 t^{2}+64 t+80$ models the height, $h$, of the ball above the ground as a function of time, $t$. Find: (a) the zeros of this function which tells us when the ball will hit the ground. (b) the time(s) the ball will be 80 feet above the ground. (c) the height the ball will be at $t=2$ seconds which is when the ball will be at its highest point.

## PRACTICE TEST

In the following exercises, factor completely.
445. $80 a^{2}+120 a^{3}$
446. $5 m(m-1)+3(m-1)$
447. $x^{2}+13 x+36$
448. $p^{2}+p q-12 q^{2}$
449. $x y-8 y+7 x-56$
450. $40 r^{2}+810$
451. $9 s^{2}-12 s+4$
452. $6 x^{2}-11 x-10$
453. $3 x^{2}-75 y^{2}$
454. $6 u^{2}+3 u-18$
455. $x^{3}+125$
456. $32 x^{5} y^{2}-162 x y^{2}$
457. $6 x^{4}-19 x^{2}+15$
458. $3 x^{3}-36 x^{2}+108 x$

In the following exercises, solve
459. $5 a^{2}+26 a=24$
461. The area of a rectangular place mat is 168 square inches. Its length is two inches longer than the width. Find the length and width of the placemat.
460. The product of two consecutive integers is 156 . Find the integers.
462. Jing is going to throw a ball from the balcony of her condo. When she throws the ball from 80 feet above the ground, the function $h(t)=-16 t^{2}+64 t+80$ models the height, $h$, of the ball above the ground as a function of time, $t$. Find: (a) the zeros of this function which tells us when the ball will hit the ground. (b) the time(s) the ball will be 128 feet above the ground. (c) the height the ball will be at $t=4$ seconds.
463. For the function, $f(x)=x^{2}-7 x+5$, (a) find when $f(x)=-7$ (b) Use this information to find two points that lie on the graph of the function.
464. For the function $f(x)=25 x^{2}-81$, find: (a) the zeros of the function (b) the $x$-intercepts of the graph of the function (c) the $y$-intercept of the graph of the function.

## $\square$

### 7.1 EXERCISES

## Practice Makes Perfect

Determine the Values for Which a Rational Expression is Undefined
In the following exercises, determine the values for which the rational expression is undefined.
1.
(a) $\frac{2 x^{2}}{z}$
(b) $\frac{4 p-1}{6 p-5}$
(c) $\frac{n-3}{n^{2}+2 n-8}$
3.
(a) $\frac{4 x^{2} y}{3 y}$
(b) $\frac{3 x-2}{2 x+1}$
(c) $\frac{u-1}{u^{2}-3 u-28}$
2.
(a) $\frac{10 m}{11 n}$
(b) $\frac{6 y+13}{4 y-9}$
(c) $\frac{b-8}{b^{2}-36}$
4.
(a) $\frac{5 p q^{2}}{9 q}$
(b) $\frac{7 a-4}{3 a+5}$
(c) $\frac{1}{x^{2}-4}$

## Simplify Rational Expressions

In the following exercises, simplify each rational expression.
5. $-\frac{44}{55}$
6. $\frac{56}{63}$
7. $\frac{8 m^{3} n}{12 m n^{2}}$
8. $\frac{36 v^{3} w^{2}}{27 v w^{3}}$
9. $\frac{8 n-96}{3 n-36}$
10. $\frac{12 p-240}{5 p-100}$
11. $\frac{x^{2}+4 x-5}{x^{2}-2 x+1}$
12. $\frac{y^{2}+3 y-4}{y^{2}-6 y+5}$
13. $\frac{a^{2}-4}{a^{2}+6 a-16}$
14. $\frac{y^{2}-2 y-3}{y^{2}-9}$
15. $\frac{p^{3}+3 p^{2}+4 p+12}{p^{2}+p-6}$
16. $\frac{x^{3}-2 x^{2}-25 x+50}{x^{2}-25}$
17. $\frac{8 b^{2}-32 b}{2 b^{2}-6 b-80}$
18. $\frac{-5 c^{2}-10 c}{-10 c^{2}+30 c+100}$
19. $\frac{3 m^{2}+30 m n+75 n^{2}}{4 m^{2}-100 n^{2}}$
20. $\frac{5 r^{2}+30 r s-35 s^{2}}{r^{2}-49 s^{2}}$
21. $\frac{a-5}{5-a}$
22. $\frac{5-d}{d-5}$
23. $\frac{20-5 y}{y^{2}-16}$
24. $\frac{4 v-32}{64-v^{2}}$
25. $\frac{w^{3}+216}{w^{2}-36}$
26. $\frac{v^{3}+125}{v^{2}-25}$
27. $\frac{z^{2}-9 z+20}{16-z^{2}}$
28. $\frac{a^{2}-5 z-36}{81-a^{2}}$

## Multiply Rational Expressions

In the following exercises, multiply the rational expressions.
29. $\frac{12}{16} \cdot \frac{4}{10}$
30. $\frac{32}{5} \cdot \frac{16}{24}$
31. $\frac{5 x^{2} y^{4}}{12 x y^{3}} \cdot \frac{6 x^{2}}{20 y^{2}}$
32. $\frac{12 a^{3} b}{b^{2}} \cdot \frac{2 a b^{2}}{9 b^{3}}$
33. $\frac{5 p^{2}}{p^{2}-5 p-36} \cdot \frac{p^{2}-16}{10 p}$
34. $\frac{3 q^{2}}{q^{2}+q-6} \cdot \frac{q^{2}-9}{9 q}$
35. $\frac{2 y^{2}-10 y}{y^{2}+10 y+25} \cdot \frac{y+5}{6 y}$
36. $\frac{z^{2}+3 z}{z^{2}-3 z-4} \cdot \frac{z-4}{z^{2}}$
37. $\frac{28-4 b}{3 b-3} \cdot \frac{b^{2}+8 b-9}{b^{2}-49}$
38. $\frac{72 m-12 m^{2}}{8 m+32} \cdot \frac{m^{2}+10 m+24}{m^{2}-36}$
39. $\frac{5 c^{2}+9 c+2}{c^{2}-25} \cdot \frac{c^{2}+10 c+25}{3 c^{2}-14 c-5}$
40. $\frac{2 d^{2}+d-3}{d^{2}-16} \cdot \frac{d^{2}-8 d+16}{2 d^{2}-9 d-18}$
41. $\frac{6 m^{2}-2 m-10}{9-m^{2}} \cdot \frac{m^{2}-6 m+9}{6 m^{2}+29 m-20}$
42. $\frac{2 n^{2}-3 n-14}{25-n^{2}} \cdot \frac{n^{2}-10 n+25}{2 n^{2}-13 n+21}$

## Divide Rational Expressions

In the following exercises, divide the rational expressions.
43. $\frac{v-5}{11-v} \div \frac{v^{2}-25}{v-11}$
44. $\frac{10+w}{w-8} \div \frac{100-w^{2}}{8-w}$
45. $\frac{3 s^{2}}{s^{2}-16} \div \frac{s^{3}-4 s^{2}+16 s}{s^{3}-64}$
46. $\frac{r^{2}-9}{15} \div \frac{r^{3}-27}{5 r^{2}+15 r+45}$
47. $\frac{p^{3}+q^{3}}{3 p^{2}+3 p q+3 q^{2}} \div \frac{p^{2}-q^{2}}{12}$
48. $\frac{v^{3}-8 w^{3}}{2 v^{2}+4 v w+8 w^{2}} \div \frac{v^{2}-4 w^{2}}{4}$
49. $\frac{x^{2}+3 x-10}{4 x} \div\left(2 x^{2}+20 x+50\right)$
50. $\frac{2 y^{2}-10 y z-48 z^{2}}{2 y-1} \div\left(4 y^{2}-32 y z\right)$
51. $\frac{\frac{2 a^{2}-a-21}{5 a+20}}{\frac{a^{2}+7 a+12}{a^{2}+8 a+16}}$
52. $\frac{\frac{3 b^{2}+2 b-8}{12 b+18}}{\frac{3 b^{2}+2 b-8}{2 b^{2}-7 b-15}}$
53. $\frac{\frac{12 c^{2}-12}{2 c^{2}-3 c+1}}{\frac{4 c+4}{6 c^{2}-13 c+5}}$
54. $\frac{\frac{4 d^{2}+7 d-2}{35 d+10}}{\frac{d^{2}-4}{7 d^{2}-12 d-4}}$
380. $\frac{x-3}{x^{2}-x-30}$

Simplify Rational Expressions
In the following exercises, simplify.
381. $\frac{18}{24}$
382. $\frac{9 m^{4}}{18 m n^{3}}$
383. $\frac{x^{2}+7 x+12}{x^{2}+8 x+16}$
384. $\frac{7 v-35}{25-v^{2}}$

Multiply Rational Expressions
In the following exercises, multiply.
385. $\frac{5}{8} \cdot \frac{4}{15}$
386. $\frac{3 x y^{2}}{8 y^{3}} \cdot \frac{16 y^{2}}{24 x}$
387.
$\frac{72 x-12 x^{2}}{8 x+32} \cdot \frac{x^{2}+10 x+24}{x^{2}-36}$
388.

$$
\frac{6 y^{2}-2 y-10}{9-y^{2}} \cdot \frac{y^{2}-6 y+9}{6 y^{2}+29 y-20}
$$

Divide Rational Expressions
In the following exercises, divide.
389. $\frac{x^{2}-4 x+12}{x^{2}+8 x+12} \div \frac{x^{2}-36}{3 x}$
390. $\frac{y^{2}-16}{4} \div \frac{y^{3}-64}{2 y^{2}+8 y+32}$
391. $\frac{11+w}{w-9} \div \frac{121-w^{2}}{9-w}$
392.
$\frac{3 y^{2}-12 y-63}{4 y+3} \div\left(6 y^{2}-42 y\right)$
393. $\frac{\frac{c^{2}-64}{3 c^{2}+26 c+16}}{\frac{c^{2}-4 c-32}{15 c+10}}$
394.
$\frac{8 a^{2}+16 a}{a-4} \cdot \frac{a^{2}+2 a-24}{a^{2}+7 a+10} \div \frac{2 a^{2}-6 a}{a+5}$

Multiply and Divide Rational Functions
395. Find $\quad R(x)=f(x) \cdot g(x) \quad$ where
$f(x)=\frac{9 x^{2}+9 x}{x^{2}-3 x-4}$ and $g(x)=\frac{x^{2}-16}{3 x^{2}+12 x}$.
396. Find $R(x)=\frac{f(x)}{g(x)}$ where $f(x)=\frac{27 x^{2}}{3 x-21}$ and $g(x)=\frac{9 x^{2}+54 x}{x^{2}-x-42}$.

### 7.2 Add and Subtract Rational Expressions

Add and Subtract Rational Expressions with a Common Denominator In the following exercises, perform the indicated operations.
397. $\frac{7}{15}+\frac{8}{15}$
398. $\frac{4 a^{2}}{2 a-1}-\frac{1}{2 a-1}$
399. $\frac{y^{2}+10 y}{y+5}+\frac{25}{y+5}$
400. $\frac{7 x^{2}}{x^{2}-9}+\frac{21 x}{x^{2}-9}$
401. $\frac{x^{2}}{x-7}-\frac{3 x+28}{x-7}$
402. $\frac{y^{2}}{y+11}-\frac{121}{y+11}$
403. $\frac{4 q^{2}-q+3}{q^{2}+6 q+5}-\frac{3 q^{2}-q-6}{q^{2}+6 q+5}$
404.
$\frac{5 t+4 t+3}{t^{2}-25}-\frac{4 t^{2}-8 t-32}{t^{2}-25}$

Add and Subtract Rational Expressions Whose Denominators Are Opposites In the following exercises, add and subtract.
405. $\frac{18 w}{6 w-1}+\frac{3 w-2}{1-6 w}$
406. $\frac{a^{2}+3 a}{a^{2}-4}-\frac{3 a-8}{4-a^{2}}$
407.
$\frac{2 b^{2}+3 b-15}{b^{2}-49}-\frac{b^{2}+16 b-1}{49-b^{2}}$
408.

$$
\frac{8 y^{2}-10 y+7}{2 y-5}+\frac{2 y^{2}+7 y+2}{5-2 y}
$$

Find the Least Common Denominator of Rational Expressions In the following exercises, find the LCD.
409. $\frac{7}{a^{2}-3 a-10}, \frac{3 a}{a^{2}-a-20}$
410. $\frac{6}{n^{2}-4}, \frac{2 n}{n^{2}-4 n+4}$
411.
$\frac{5}{3 p^{2}+17 p-6}, \frac{2 m}{3 p^{2}-23 p-8}$

Add and Subtract Rational Expressions with Unlike Denominators In the following exercises, perform the indicated operations.
412. $\frac{7}{5 a}+\frac{3}{2 b}$
413. $\frac{2}{c-2}+\frac{9}{c+3}$
414. $\frac{3 x}{x^{2}-9}+\frac{5}{x^{2}+6 x+9}$
415.
$\frac{2 x}{x^{2}+10 x+24}+\frac{3 x}{x^{2}+8 x+16}$
416. $\frac{5 q}{p^{2} q-p^{2}}+\frac{4 q}{q^{2}-1}$
417. $\frac{3 y}{y+2}-\frac{y+2}{y+8}$
418. $\frac{-3 w-15}{w^{2}+w-20}-\frac{w+2}{4-w}$
419. $\frac{7 m+3}{m+2}-5$
420. $\frac{n}{n+3}+\frac{2}{n-3}-\frac{n-9}{n^{2}-9}$
421. $\frac{8 a}{a^{2}-64}-\frac{4}{a+8}$
422. $\frac{5}{12 x^{2} y}+\frac{7}{20 x y^{3}}$

Add and Subtract Rational Functions
In the following exercises, find $R(x)=f(x)+g(x)$ where $f(x)$ and $g(x)$ are given.
423. $f(x)=\frac{2 x^{2}+12 x-11}{x^{2}+3 x-10}, g(x)=\frac{x+1}{2-x}$
424. $f(x)=\frac{-4 x+31}{x^{2}+x-30}, \quad g(x)=\frac{5}{x+6}$

In the following exercises, find $R(x)=f(x)-g(x)$ where $f(x)$ and $g(x)$ are given.
425. $f(x)=\frac{4 x}{x^{2}-121}, \quad g(x)=\frac{2}{x-11}$
426. $f(x)=\frac{7}{x+6}, \quad g(x)=\frac{14 x}{x^{2}-36}$

### 7.3 Simplify Complex Rational Expressions

Simplify a Complex Rational Expression by Writing It as Division In the following exercises, simplify.
427. $\frac{\frac{7 x}{x+2}}{\frac{14 x^{2}}{x^{2}-4}}$
428. $\frac{\frac{2}{5}+\frac{5}{6}}{\frac{1}{3}+\frac{1}{4}}$
429. $\frac{x-\frac{3 x}{x+5}}{\frac{1}{x+5}+\frac{1}{x-5}}$
430. $\frac{\frac{2}{m}+\frac{m}{n}}{\frac{n}{m}-\frac{1}{n}}$

Simplify a Complex Rational Expression by Using the LCD
In the following exercises, simplify.
431. $\frac{\frac{1}{3}+\frac{1}{8}}{\frac{1}{4}+\frac{1}{12}}$
432. $\frac{\frac{3}{a^{2}}-\frac{1}{b}}{\frac{1}{a}+\frac{1}{b^{2}}}$
433. $\frac{\frac{2}{z^{2}-49}+\frac{1}{z+7}}{\frac{9}{z+7}+\frac{12}{z-7}}$
434. $\frac{\frac{3}{y^{2}-4 y-32}}{\frac{2}{y-8}+\frac{1}{y+4}}$

### 7.4 7.4 Solve Rational Equations

Solve Rational Equations
In the following exercises, solve.
435. $\frac{1}{2}+\frac{2}{3}=\frac{1}{x}$
436. $1-\frac{2}{m}=\frac{8}{m^{2}}$
437. $\frac{1}{b-2}+\frac{1}{b+2}=\frac{3}{b^{2}-4}$
438. $\frac{3}{q+8}-\frac{2}{q-2}=1$
439.
$\frac{v-15}{v^{2}-9 v+18}=\frac{4}{v-3}+\frac{2}{v-6}$
440. $\frac{z}{12}+\frac{z+3}{3 z}=\frac{1}{z}$

Solve Rational Equations that Involve Functions
441. For rational function, $f(x)=\frac{x+2}{x^{2}-6 x+8}$, (a) 442. For rational function, $f(x)=\frac{2-x}{x^{2}+7 x+10}$,
find the domain of the function (b) solve $f(x)=1$ (c) find the domain of the function (b) solve $f(x)=2$ (c) find the points on the graph at this function value. find the points on the graph at this function value.

Solve a Rational Equation for a Specific Variable
In the following exercises, solve for the indicated variable.
443. $\frac{V}{l}=h w$ for $l$.
444. $\frac{1}{x}-\frac{2}{y}=5$ for $y$.
445. $x=\frac{y+5}{z-7}$ for $z$.
446. $P=\frac{k}{V}$ for $V$.
460. Curtis was training for a triathlon. He ran 8 kilometers and biked 32 kilometers in a total of 3 hours. His running speed was 8 kilometers per hour less than his biking speed. What was his running speed?

## Solve Work Applications

In the following exercises, solve.
461. Brandy can frame a room in 1 hour, while Jake takes 4 hours. How long could they frame a room working together?
464. Marta and Deb work together writing a book that takes them 90 days. If Sue worked alone it would take her 120 days. How long would it take Deb to write the book alone?

Solve Direct Variation Problems In the following exercises, solve
465. If $y$ varies directly as $x$ when $y=9$ and $x=3$, find $x$ when $y=21$.
468. If the cost of a pizza varies directly with its diameter, and if an 8 " diameter pizza costs $\$ 12$, how much would a $6^{\prime \prime}$ diameter pizza cost?
462. Prem takes 3 hours to mow the lawn while her cousin, Barb, takes 2 hours. How long will it take them working together?
463. Jeffrey can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take the helper to paint the house alone?
467. Vanessa is traveling to see her fiancé. The distance, $d$, varies directly with the speed, $v$, she drives. If she travels 258 miles driving 60 mph , how far would she travel going 70 mph ?
469. The distance to stop a car varies directly with the square of its speed. It takes 200 feet to stop a car going 50 mph . How many feet would it take to stop a car going 60 mph?
471. The number of tickets for a music fundraiser varies inversely with the price of the tickets. If Madelyn has just enough money to purchase 12 tickets for $\$ 6$, how many tickets can Madelyn afford to buy if the price increased to $\$ 8$ ?
472. On a string instrument, the length of a string varies inversely with the frequency of its vibrations. If an 11 -inch string on a violin has a frequency of 360 cycles per second, what frequency does a 12 -inch string have?

### 7.6 Solve Rational Inequalities

Solve Rational Inequalities
In the following exercises, solve each rational inequality and write the solution in interval notation.
473. $\frac{x-3}{x+4} \leq 0$
474. $\frac{5 x}{x-2}>1$
475. $\frac{3 x-2}{x-4} \leq 2$
476. $\frac{1}{x^{2}-4 x-12}<0$
477. $\frac{1}{2}-\frac{4}{x^{2}} \geq \frac{1}{x}$
478. $\frac{4}{x-2}<\frac{3}{x+1}$

## PRACTICE TEST

In the following exercises, simplify.
483. $\frac{4 a^{2} b}{12 a b^{2}}$
484. $\frac{6 x-18}{x^{2}-9}$

In the following exercises, perform the indicated operation and simplify.
485. $\frac{4 x}{x+2} \cdot \frac{x^{2}+5 x+6}{12 x^{2}}$
486. $\frac{2 y^{2}}{y^{2}-1} \div \frac{y^{3}-y^{2}+y}{y^{3}-1}$
487.
489.
490.
488. $\frac{-3 a}{3 a-3}+\frac{5 a}{a^{2}+3 a-4}$
$\frac{2 n^{2}+8 n-1}{n^{2}-1}-\frac{n^{2}-7 n-1}{1-n^{2}}$

$$
\frac{10 x^{2}+16 x-7}{8 x-3}+\frac{2 x^{2}+3 x-1}{3-8 x}
$$

491. $\frac{\frac{1}{m}-\frac{1}{n}}{\frac{1}{n}+\frac{1}{m}}$

In the following exercises, solve each equation.
492. $\frac{1}{x}+\frac{3}{4}=\frac{5}{8}$
493. $\frac{1}{z-5}+\frac{1}{z+5}=\frac{1}{z^{2}-25}$
494.
$\frac{z}{2 z+8}-\frac{3}{4 z-8}=\frac{3 z^{2}-16 z-16}{8 z^{2}+2 z-64}$

In the following exercises, solve each rational inequality and write the solution in interval notation.
495. $\frac{6 x}{x-6} \leq 2$
496. $\frac{2 x+3}{x-6}>1$
497. $\frac{1}{2}+\frac{12}{x^{2}} \geq \frac{5}{x}$

In the following exercises, find $R(x)$ given $f(x)=\frac{x-4}{x^{2}-3 x-10}$ and $g(x)=\frac{x-5}{x^{2}-2 x-8}$.
498. $R(x)=f(x)-g(x)$
499. $R(x)=f(x) \cdot g(x)$
500. $R(x)=f(x) \div g(x)$
501. Given the function,
$R(x)=\frac{2}{2 x^{2}+x-15}$, find the
values of $x$ that make the function less than or equal to 0 .

In the following exercises, solve.
502. If $y$ varies directly with $x$, and $x=5$ when $y=30$, find $x$ when $y=42$.
503. If $y$ varies inversely with the square of $x$ and $x=3$ when $y=9$, find $y$ when $x=4$.
504. Matheus can ride his bike for 30 miles with the wind in the same amount of time that he can go 21 miles against the wind. If the wind's speed is 6 mph , what is Matheus' speed on his bike?

### 8.6 EXERCISES

## Practice Makes Perfect

## Solve Radical Equations

In the following exercises, solve.
287. $\sqrt{5 x-6}=8$
290. $\sqrt{3 y-4}=-2$
293. $\sqrt{2 m-3}-5=0$
296. $\sqrt{12 u+1}-11=0$
299. $\sqrt{2 u-3}+2=0$
302. $\sqrt{v-10}+10=v$
305. $\sqrt[3]{6 x+4}=4$
308. $\sqrt[3]{9 x-1}-1=-5$
311. $(8 x+5)^{\frac{1}{3}}+2=-1$
314. $(5 x-4)^{\frac{1}{4}}+7=9$
317. $\sqrt{z+100}-z=-10$
320. $2 \sqrt{5 x+1}-8=0$
288. $\sqrt{4 x-3}=7$
291. $\sqrt[3]{2 x}=-2$
294. $\sqrt{2 n-1}-3=0$
297. $\sqrt{4 m+2}+2=6$
300. $\sqrt{5 v-2}+5=0$
303. $\sqrt{r-1}=r-1$
306. $\sqrt[3]{11 x+4}=5$
309. $(6 x+1)^{\frac{1}{2}}-3=4$
312. $(12 x-5)^{\frac{1}{3}}+8=3$
315. $\sqrt{x+1}-x+1=0$
318. $\sqrt{w+25}-w=-5$
321. $2 \sqrt{8 r+1}-8=2$

Solve Radical Equations with Two Radicals
In the following exercises, solve.
323. $\sqrt{3 u+7}=\sqrt{5 u+1}$
324. $\sqrt{4 v+1}=\sqrt{3 v+3}$
325. $\sqrt{8+2 r}=\sqrt{3 r+10}$
326. $\sqrt{10+2 c}=\sqrt{4 c+16}$
327. $\sqrt[3]{5 x-1}=\sqrt[3]{x+3}$
330.
329.
$\sqrt[3]{2 x^{2}+9 x-18}=\sqrt[3]{x^{2}+3 x-2}$
$\sqrt[3]{x^{2}-x+18}=\sqrt[3]{2 x^{2}-3 x-6}$
332. $\sqrt{r}+6=\sqrt{r+8}$
333. $\sqrt{u}+1=\sqrt{u+4}$
334. $\sqrt{x}+1=\sqrt{x+2}$
335. $\sqrt{a+5}-\sqrt{a}=1$
336. $-2=\sqrt{d-20}-\sqrt{d}$
339. $\sqrt{2 x-1}-\sqrt{x-1}=1$
338. $\sqrt{3 x+1}=1+\sqrt{2 x-1}$
328. $\sqrt[3]{8 x-5}=\sqrt[3]{3 x+5}$
331. $\sqrt{a}+2=\sqrt{a+4}$
337. $\sqrt{2 x+1}=1+\sqrt{x}$
340. $\sqrt{x+1}-\sqrt{x-2}=1$
289. $\sqrt{5 x+1}=-3$
292. $\sqrt[3]{4 x-1}=3$
295. $\sqrt{6 v-2}-10=0$
298. $\sqrt{6 n+1}+4=8$
301. $\sqrt{u-3}-3=u$
304. $\sqrt{s-8}=s-8$
307. $\sqrt[3]{4 x+5}-2=-5$
310. $(3 x-2)^{\frac{1}{2}}+1=6$
313. $(12 x-3)^{\frac{1}{4}}-5=-2$
316. $\sqrt{y+4}-y+2=0$
319. $3 \sqrt{2 x-3}-20=7$
322. $3 \sqrt{7 y+1}-10=8$

## REVIEW EXERCISES

### 8.1 Simplify Expressions with Roots

Simplify Expressions with Roots
In the following exercises, simplify.
481. (a) $\sqrt{225}$ (b) $-\sqrt{16}$
482. (a) $-\sqrt{169}$ (b) $\sqrt{-8}$
483. (a) $\sqrt[3]{8}$ (b) $\sqrt[4]{81}$ © $\sqrt[5]{243}$
484.
(a) $\sqrt[3]{-512}$ (b) $\sqrt[4]{-81}$ © $\sqrt[5]{-1}$

Estimate and Approximate Roots
In the following exercises, estimate each root between two consecutive whole numbers.
485. (a) $\sqrt{68}$ (b) $\sqrt[3]{84}$

In the following exercises, approximate each root and round to two decimal places.
486. (a) $\sqrt{37}$ (b) $\sqrt[3]{84}$ (c) $\sqrt[4]{125}$

Simplify Variable Expressions with Roots
In the following exercises, simplify using absolute values as necessary.

## 487.

(a) $\sqrt[3]{a^{3}}$
(b) $\sqrt[7]{b^{7}}$
490.
(a) $\sqrt{121 m^{20}}$
(b) $-\sqrt{64 a^{2}}$
488.
(a) $\sqrt{a^{14}}$
(b) $\sqrt{w^{24}}$
489.
(a) $\sqrt[4]{m^{8}}$
(b) $\sqrt[5]{n^{20}}$
492.
(a) $\sqrt{144 x^{2} y^{2}}$
(b) $\sqrt{169 w^{8} y^{10}}$
(C) $\sqrt[3]{8 a^{51} b^{6}}$

### 8.2 Simplify Radical Expressions

Use the Product Property to Simplify Radical Expressions
In the following exercises, use the Product Property to simplify radical expressions.
493. $\sqrt{125}$
494. $\sqrt{675}$
495. (a) $\sqrt[3]{625}$ (b) $\sqrt[6]{128}$

In the following exercises, simplify using absolute value signs as needed.
496.
(a) $\sqrt{a^{23}}$
(b) $\sqrt[3]{b^{8}}$
(c) $\sqrt[8]{c^{13}}$
499.
(a) $\sqrt[5]{-32}$
(b) $\sqrt[8]{-1}$
497.
(a) $\sqrt{80 s^{15}}$
(b) $\sqrt[5]{96 a^{7}}$
(c) $\sqrt[6]{128 b^{7}}$
498.
(a) $\sqrt{96 r^{3} s^{3}}$
(b) $\sqrt[3]{80 x^{7} y^{6}}$
(c) $\sqrt[4]{80 x^{8} y^{9}}$
500.
(a) $8+\sqrt{96}$
(b) $\frac{2+\sqrt{40}}{2}$

## Use the Quotient Property to Simplify Radical Expressions

In the following exercises, use the Quotient Property to simplify square roots.
501. (a) $\sqrt{\frac{72}{98}}$ (b) $\sqrt[3]{\frac{24}{81}}$ (c) $\sqrt[4]{\frac{6}{96}}$
502. (a) $\sqrt{\frac{y^{4}}{y^{8}}}$ (b) $\sqrt[5]{\frac{u^{21}}{u^{11}}}$ (c) $\sqrt[6]{\frac{v^{30}}{v^{12}}}$
503. $\sqrt{\frac{300 m^{5}}{64}}$
504.
(a) $\sqrt{\frac{28 p^{7}}{q^{2}}}$
(b) $\sqrt[3]{\frac{81 s^{8}}{t^{3}}}$
(c) $\sqrt[4]{\frac{64 p^{15}}{q^{12}}}$
505.
(a) $\sqrt{\frac{27 p^{2} q}{108 p^{4} q^{3}}}$
(b) $\sqrt[3]{\frac{16 c^{5} d^{7}}{250 c^{2} d^{2}}}$
(c) $\sqrt[6]{\frac{2 m^{9} n^{7}}{128 m^{3} n}}$
506.
(a) $\frac{\sqrt{80 q^{5}}}{\sqrt{5 q}}$
(b) $\frac{\sqrt[3]{-625}}{\sqrt[3]{5}}$
(c) $\frac{\sqrt[4]{80 m^{7}}}{\sqrt[4]{5 m}}$

### 8.3 Simplify Rational Exponents

Simplify expressions with $a^{\frac{1}{n}}$
In the following exercises, write as a radical expression.
507. (a) $r^{\frac{1}{2}}$ (b) $s^{\frac{1}{3}}$ (c) $t^{\frac{1}{4}}$

In the following exercises, write with a rational exponent.
508. (a) $\sqrt{21 p}$ (b) $\sqrt[4]{8 q}$ © $4 \sqrt[6]{36 r}$

In the following exercises, simplify.
509.
(a) $625^{\frac{1}{4}}$
510.
(a) $(-1,000)^{\frac{1}{3}}$
(b) $243^{\frac{1}{5}}$
(b) $-1,000^{\frac{1}{3}}$
(c) $(1,000)^{-\frac{1}{3}}$
511.
(a) $(-32)^{\frac{1}{5}}$
(b) $(243)^{-\frac{1}{5}}$
(C) $-125^{\frac{1}{3}}$

Simplify Expressions with $a^{\frac{m}{n}}$
In the following exercises, write with a rational exponent.
512.
(a) $\sqrt[4]{r^{7}}$
(b) $(\sqrt[5]{2 p q})^{3}$
(c) $\sqrt[4]{\left(\frac{12 m}{7 n}\right)^{3}}$

In the following exercises, simplify.
513.
(a) $25^{\frac{3}{2}}$
(b) $9^{-\frac{3}{2}}$
(c) $(-64)^{\frac{2}{3}}$
514.
(b) $-64^{-\frac{3}{2}}$
(a) $-64^{\frac{3}{2}}$
(c) $(-64)^{\frac{3}{2}}$

Use the Laws of Exponents to Simplify Expressions with Rational Exponents In the following exercises, simplify.
515.
(a) $6^{\frac{5}{2}} \cdot 6^{\frac{1}{2}}$
(b) $\left(b^{15}\right)^{\frac{3}{5}}$
(C) $\frac{w^{\frac{2}{7}}}{w^{\frac{9}{7}}}$

### 8.4 Add, Subtract and Multiply Radical Expressions

Add and Subtract Radical Expressions
In the following exercises, simplify.
516.
(a) $\frac{a^{\frac{3}{4}} \cdot a^{-\frac{1}{4}}}{a^{-\frac{10}{4}}}$
(b) $\left(\frac{27 b^{\frac{2}{3}} c^{-\frac{5}{2}}}{b^{-\frac{7}{3}} c^{\frac{1}{2}}}\right)^{\frac{1}{3}}$
517.
(a) $7 \sqrt{2}-3 \sqrt{2}$
(b) $7 \sqrt[3]{p}+2 \sqrt[3]{p}$
(c) $5 \sqrt[3]{x}-3 \sqrt[3]{x}$
517.
518.
(a) $\sqrt{11 b}-5 \sqrt{11 b}+3 \sqrt{11 b}$
(b) $8 \sqrt[4]{11 c d}+5 \sqrt[4]{11 c d}-9 \sqrt[4]{11 c d}$
(a) $\sqrt{48}+\sqrt{27}$
(b) $\sqrt[3]{54}+\sqrt[3]{128}$
(c) $6 \sqrt[4]{5}-\frac{3}{2} \sqrt[4]{320}$

## 520.

(a) $\sqrt{80 c^{7}}-\sqrt{20 c^{7}}$
521. $3 \sqrt{75 y^{2}}+8 y \sqrt{48}-\sqrt{300 y^{2}}$
(b) $2 \sqrt[4]{162 r^{10}}+4 \sqrt[4]{32 r^{10}}$

Multiply Radical Expressions
In the following exercises, simplify.
522.
(a) $(5 \sqrt{6})(-\sqrt{12})$
(b) $(-2 \sqrt[4]{18})(-\sqrt[4]{9})$
523.
(a) $\left(3 \sqrt{2 x^{3}}\right)\left(7 \sqrt{18 x^{2}}\right)$
(b) $\left(-6 \sqrt[3]{20 a^{2}}\right)\left(-2 \sqrt[3]{16 a^{3}}\right)$

Use Polynomial Multiplication to Multiply Radical Expressions In the following exercises, multiply.
524.
(a) $\sqrt{11}(8+4 \sqrt{11})$
(b) $\sqrt[3]{3}(\sqrt[3]{9}+\sqrt[3]{18})$
525.
(a) $(3-2 \sqrt{7})(5-4 \sqrt{7})$
(b) $(\sqrt[3]{x}-5)(\sqrt[3]{x}-3)$
526. $(2 \sqrt{7}-5 \sqrt{11})(4 \sqrt{7}+9 \sqrt{11})$
527.
(a) $(4+\sqrt{11})^{2}$
(b) $(3-2 \sqrt{5})^{2}$

### 8.5 Divide Radical Expressions

Divide Square Roots
In the following exercises, simplify.
530.
(a) $\frac{\sqrt{48}}{\sqrt{75}}$
(b) $\frac{\sqrt[3]{81}}{\sqrt[3]{24}}$
(a) $\frac{\sqrt{320 m n^{-5}}}{\sqrt{45 m^{-7} n^{3}}}$
(b) $\frac{\sqrt[3]{16 x^{4} y^{-2}}}{\sqrt[3]{-54 x^{-2} y^{4}}}$

Rationalize a One Term Denominator
In the following exercises, rationalize the denominator.
532. (a) $\frac{8}{\sqrt{3}}$ (b) $\sqrt{\frac{7}{40}}$ (c) $\frac{8}{\sqrt{2 y}}$
533. (a) $\frac{1}{\sqrt[3]{11}}$ (b) $\sqrt[3]{\frac{7}{54}}$ (c) $\frac{3}{\sqrt[3]{3 x^{2}}}$

Rationalize a Two Term Denominator
In the following exercises, simplify.
535. $\frac{7}{2-\sqrt{6}}$
536. $\frac{\sqrt{5}}{\sqrt{n}-\sqrt{7}}$
537. $\frac{\sqrt{x}+\sqrt{8}}{\sqrt{x}-\sqrt{8}}$

### 8.6 Solve Radical Equations

Solve Radical Equations
In the following exercises, solve.
538. $\sqrt{4 x-3}=7$
539. $\sqrt{5 x+1}=-3$
541. $\sqrt{u-3}+3=u$
542. $\sqrt[3]{4 x+5}-2=-5$
543. $(8 x+5)^{\frac{1}{3}}+2=-1$
544. $\sqrt{y+4}-y+2=0$
545. $2 \sqrt{8 r+1}-8=2$

Solve Radical Equations with Two Radicals In the following exercises, solve.
546. $\sqrt{10+2 c}=\sqrt{4 c+16}$
547.
$\sqrt[3]{2 x^{2}+9 x-18}=\sqrt[3]{x^{2}+3 x-2}$
548. $\sqrt{r}+6=\sqrt{r+8}$
549. $\sqrt{x+1}-\sqrt{x-2}=1$

## Use Radicals in Applications

In the following exercises, solve. Round approximations to one decimal place.
550. Landscaping Reed wants to have a square garden plot in his backyard. He has enough compost to cover an area of 75 square feet. Use the formula $s=\sqrt{A}$ to find the length of each side of his garden. Round your answer to the nearest tenth of a foot.
551. Accident investigation An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 175 feet. Use the formula $s=\sqrt{24 d}$ to find the speed of the vehicle before the brakes were applied. Round your answer to the nearest tenth.

### 8.7 Use Radicals in Functions

Evaluate a Radical Function
In the following exercises, evaluate each function.
552. $g(x)=\sqrt{6 x+1}$, find
553. $G(x)=\sqrt{5 x-1}$, find
(a) $g(4)$
(a) $G(5)$
(b) $g(8)$
(b) $G(2)$
554. $h(x)=\sqrt[3]{x^{2}-4}$, find
(a) $h(-2)$
(b) $h(6)$
555. For the function
$g(x)=\sqrt[4]{4-4 x}$, find
(a) $g(1)$
(b) $g(-3)$

Find the Domain of a Radical Function
In the following exercises, find the domain of the function and write the domain in interval notation.
556. $g(x)=\sqrt{2-3 x}$
557. $F(x)=\sqrt{\frac{x+3}{x-2}}$
558. $f(x)=\sqrt[3]{4 x^{2}-16}$
559. $F(x)=\sqrt[4]{10-7 x}$

## Graph Radical Functions

In the following exercises, (a) find the domain of the function (b) graph the function (c) use the graph to determine the range.
560. $g(x)=\sqrt{x+4}$
561. $g(x)=2 \sqrt{x}$
562. $f(x)=\sqrt[3]{x-1}$
563. $f(x)=\sqrt[3]{x}+3$

### 8.8 Use the Complex Number System

## Evaluate the Square Root of a Negative Number

In the following exercises, write each expression in terms of $i$ and simplify if possible.

## 564.

(a) $\sqrt{-100}$
(b) $\sqrt{-13}$
(c) $\sqrt{-45}$

Add or Subtract Complex Numbers
In the following exercises, add or subtract.
565. $\sqrt{-50}+\sqrt{-18}$
566. $(8-i)+(6+3 i)$
567. $(6+i)-(-2-4 i)$
568.
$(-7-\sqrt{-50})-(-32-\sqrt{-18})$

Multiply Complex Numbers
In the following exercises, multiply.
569. $(-2-5 i)(-4+3 i)$
570. $-6 i(-3-2 i)$
571. $\sqrt{-4} \cdot \sqrt{-16}$
572. $(5-\sqrt{-12})(-3+\sqrt{-75})$

In the following exercises, multiply using the Product of Binomial Squares Pattern.
573. $(-2-3 i)^{2}$

In the following exercises, multiply using the Product of Complex Conjugates Pattern.
574. $(9-2 i)(9+2 i)$

## Divide Complex Numbers

In the following exercises, divide.
575. $\frac{2+i}{3-4 i}$
576. $\frac{-4}{3-2 i}$

Simplify Powers of $\boldsymbol{i}$
In the following exercises, simplify.
577. $i^{48}$
578. $i^{255}$

## PRACTICE TEST

In the following exercises, simplify using absolute values as necessary.
579. $\sqrt[3]{125 x^{9}}$
580. $\sqrt{169 x^{8} y^{6}}$
581. $\sqrt[3]{72 x^{8} y^{4}}$
582. $\sqrt{\frac{45 x^{3} y^{4}}{180 x^{5} y^{2}}}$

In the following exercises, simplify. Assume all variables are positive.
583. (a) $216^{-\frac{1}{4}}$ (b) $-49^{\frac{3}{2}}$
584. $\sqrt{-45}$
585. $\frac{x^{-\frac{1}{4}} \cdot x^{\frac{5}{4}}}{x^{-\frac{3}{4}}}$
586. $\left(\frac{8 x^{\frac{2}{3}} y^{-\frac{5}{2}}}{x^{-\frac{7}{3}} y^{\frac{1}{2}}}\right)^{\frac{1}{3}}$
587. $\sqrt{48 x^{5}}-\sqrt{75 x^{5}}$
588. $\sqrt{27 x^{2}}-4 x \sqrt{12}+\sqrt{108 x^{2}}$
589. $2 \sqrt{12 x^{5}} \cdot 3 \sqrt{6 x^{3}}$
590. $\sqrt[3]{4}(\sqrt[3]{16}-\sqrt[3]{6})$
591. $(4-3 \sqrt{3})(5+2 \sqrt{3})$
592. $\frac{\sqrt[3]{128}}{\sqrt[3]{54}}$
593. $\frac{\sqrt{245 x y^{-4}}}{\sqrt{45 x^{-4} y^{3}}}$
594. $\frac{1}{\sqrt[3]{5}}$
595. $\frac{3}{2+\sqrt{3}}$
596. $\sqrt{-4} \cdot \sqrt{-9}$
597. $-4 i(-2-3 i)$
598. $\frac{4+i}{3-2 i}$
599. $i^{172}$

In the following exercises, solve.
600. $\sqrt{2 x+5}+8=6$
601. $\sqrt{x+5}+1=x$
602.
$\sqrt[3]{2 x^{2}-6 x-23}=\sqrt[3]{x^{2}-3 x+5}$

In the following exercise, (a) find the domain of the function (b) graph the function (c) use the graph to determine the range.
603. $g(x)=\sqrt{x+2}$

### 9.8 Solve Quadratic Inequalities

- Solve a Quadratic Inequality Graphically

Step 1. Write the quadratic inequality in standard form.
Step 2. Graph the function $f(x)=a x^{2}+b x+c$ using properties or transformations.
Step 3. Determine the solution from the graph.

- How to Solve a Quadratic Inequality Algebraically

Step 1. Write the quadratic inequality in standard form.
Step 2. Determine the critical points -- the solutions to the related quadratic equation.
Step 3. Use the critical points to divide the number line into intervals.
Step 4. Above the number line show the sign of each quadratic expression using test points from each interval substituted into the original inequality.
Step 5. Determine the intervals where the inequality is correct. Write the solution in interval notation.

## REVIEW EXERCISES

### 9.1 Solve Quadratic Equations Using the Square Root Property

Solve Quadratic Equations of the form $a x^{2}=k$ Using the Square Root Property In the following exercises, solve using the Square Root Property.
395. $y^{2}=144$
396. $n^{2}-80=0$
397. $4 a^{2}=100$
398. $2 b^{2}=72$
399. $r^{2}+32=0$
400. $t^{2}+18=0$
401. $\frac{2}{3} w^{2}-20=30$
402. 11. $5 c^{2}+3=19$

Solve Quadratic Equations of the Form $a(x-h)^{2}=k$ Using the Square Root Property In the following exercises, solve using the Square Root Property.
403. $(p-5)^{2}+3=19$
404. $(u+1)^{2}=45$
405. $\left(x-\frac{1}{4}\right)^{2}=\frac{3}{16}$
406. $\left(y-\frac{2}{3}\right)^{2}=\frac{2}{9}$
407. $(n-4)^{2}-50=150$
408. $(4 c-1)^{2}=-18$
409. $n^{2}+10 n+25=12$
410. $64 a^{2}+48 a+9=81$

### 9.2 Solve Quadratic Equations by Completing the Square

Solve Quadratic Equations Using Completing the Square
In the following exercises, complete the square to make a perfect square trinomial. Then write the result as a binomial squared.
411. $x^{2}+22 x$
412. $m^{2}-8 m$
413. $a^{2}-3 a$
414. $b^{2}+13 b$

In the following exercises, solve by completing the square.
415. $d^{2}+14 d=-13$
416. $y^{2}-6 y=36$
417. $m^{2}+6 m=-109$
418. $t^{2}-12 t=-40$
419. $v^{2}-14 v=-31$
420. $w^{2}-20 w=100$
421. $m^{2}+10 m-4=-13$
422. $n^{2}-6 n+11=34$
423. $a^{2}=3 a+8$
424. $b^{2}=11 b-5$
425. $(u+8)(u+4)=14$
426. $(z-10)(z+2)=28$

Solve Quadratic Equations of the form $a x^{2}+b x+c=0$ by Completing the Square In the following exercises, solve by completing the square.
427. $3 p^{2}-18 p+15=15$
428. $5 q^{2}+70 q+20=0$
429. $4 y^{2}-6 y=4$
430. $2 x^{2}+2 x=4$
431. $3 c^{2}+2 c=9$
432. $4 d^{2}-2 d=8$
433. $2 x^{2}+6 x=-5$
434. $2 x^{2}+4 x=-5$

### 9.3 Solve Quadratic Equations Using the Quadratic Formula

In the following exercises, solve by using the Quadratic Formula.
435. $4 x^{2}-5 x+1=0$
436. $7 y^{2}+4 y-3=0$
437. $r^{2}-r-42=0$
438. $t^{2}+13 t+22=0$
439. $4 v^{2}+v-5=0$
440. $2 w^{2}+9 w+2=0$
441. $3 m^{2}+8 m+2=0$
442. $5 n^{2}+2 n-1=0$
443. $6 a^{2}-5 a+2=0$
444. $4 b^{2}-b+8=0$
445. $u(u-10)+3=0$
446. $5 z(z-2)=3$
447. $\frac{1}{8} p^{2}-\frac{1}{5} p=-\frac{1}{20}$
448. $\frac{2}{5} q^{2}+\frac{3}{10} q=\frac{1}{10}$
449. $4 c^{2}+4 c+1=0$
450. $9 d^{2}-12 d=-4$

Use the Discriminant to Predict the Number of Solutions of a Quadratic Equation
In the following exercises, determine the number of solutions for each quadratic equation.
451.
452.
(a) $9 x^{2}-6 x+1=0$
(a) $5 x^{2}-7 x-8=0$
(b) $3 y^{2}-8 y+1=0$
(b) $7 x^{2}-10 x+5=0$
(c) $7 m^{2}+12 m+4=0$
(c) $25 x^{2}-90 x+81=0$
(d) $5 n^{2}-n+1=0$
(d) $15 x^{2}-8 x+4=0$

Identify the Most Appropriate Method to Use to Solve a Quadratic Equation
In the following exercises, identify the most appropriate method (Factoring, Square Root, or Quadratic Formula) to use to solve each quadratic equation. Do not solve.
453.
(a) $16 r^{2}-8 r+1=0$
(b) $5 t^{2}-8 t+3=9$
(c) $3(c+2)^{2}=15$
454.
(a) $4 d^{2}+10 d-5=21$
(b) $25 x^{2}-60 x+36=0$
(C) $6(5 v-7)^{2}=150$

### 9.6 Graph Quadratic Functions Using Properties

Recognize the Graph of a Quadratic Function
In the following exercises, graph by plotting point.
476. Graph $y=x^{2}-2$
477. Graph $y=-x^{2}+3$

In the following exercises, determine if the following parabolas open up or down.
478.
479.
(a) $y=-3 x^{2}+3 x-1$
(a) $y=x^{2}+8 x-1$
(b) $y=5 x^{2}+6 x+3$
(b) $y=-4 x^{2}-7 x+1$

Find the Axis of Symmetry and Vertex of a Parabola
In the following exercises, find (a) the equation of the axis of symmetry and (b) the vertex.
480. $y=-x^{2}+6 x+8$
481. $y=2 x^{2}-8 x+1$

Find the Intercepts of a Parabola
In the following exercises, find the $x$ - and $y$-intercepts.
482. $y=x^{2}-4 x+5$
483. $y=x^{2}-8 x+15$
484. $y=x^{2}-4 x+10$
485. $y=-5 x^{2}-30 x-46$
486. $y=16 x^{2}-8 x+1$
487. $y=x^{2}+16 x+64$

## Graph Quadratic Functions Using Properties

In the following exercises, graph by using its properties.
488. $y=x^{2}+8 x+15$
489. $y=x^{2}-2 x-3$
490. $y=-x^{2}+8 x-16$
491. $y=4 x^{2}-4 x+1$
492. $y=x^{2}+6 x+13$
493. $y=-2 x^{2}-8 x-12$

Solve Maximum and Minimum Applications
In the following exercises, find the minimum or maximum value.
494. $y=7 x^{2}+14 x+6$
495. $y=-3 x^{2}+12 x-10$

## PRACTICE TEST

529. Use the Square Root Property to solve the quadratic equation $3(w+5)^{2}=27$.
530. Use Completing the Square to solve the quadratic equation $a^{2}-8 a+7=23$.
531. Use the Quadratic Formula to solve the quadratic equation $2 m^{2}-5 m+3=0$.

Solve the following quadratic equations. Use any method.
532. $2 x(3 x-2)-1=0$
533. $\frac{9}{4} y^{2}-3 y+1=0$

Use the discriminant to determine the number and type of solutions of each quadratic equation.
534. $6 p^{2}-13 p+7=0$
535. $3 q^{2}-10 q+12=0$

Solve each equation.
536. $4 x^{4}-17 x^{2}+4=0$
537. $y^{\frac{2}{3}}+2 y^{\frac{1}{3}}-3=0$

For each parabola, find (a) which direction it opens, (b) the equation of the axis of symmetry, (c) the vertex, (d) the $x$ - and $y$-intercepts, and e) the maximum or minimum value.
538. $y=3 x^{2}+6 x+8$
539. $y=-x^{2}-8 x+16$

Graph each quadratic function using intercepts, the vertex, and the equation of the axis of symmetry.
540. $f(x)=x^{2}+6 x+9$
541. $f(x)=-2 x^{2}+8 x+4$

In the following exercises, graph each function using transformations.
542. $f(x)=(x+3)^{2}+2$
543. $f(x)=x^{2}-4 x-1$

In the following exercises, solve each inequality algebraically and write any solution in interval notation.
544. $x^{2}-6 x-8 \leq 0$
545. $2 x^{2}+x-10>0$

Model the situation with a quadratic equation and solve by any method.
546. Find two consecutive even 547. The length of a diagonal of numbers whose product is 360 .
a rectangle is three more than the width. The length of the rectangle is three times the width. Find the length of the diagonal. (Round to the nearest tenth.)
548. A water balloon is launched upward at the rate of $86 \mathrm{ft} / \mathrm{sec}$. Using the formula $h=-16 t^{2}+86 t$ find how long it will take the balloon to reach the maximum height, and then find the maximum height. Round to the nearest tenth.

### 11.1 EXERCISES <br> Practice Makes Perfect

## Use the Distance Formula

In the following exercises, find the distance between the points. Write the answer in exact form and then find the decimal approximation, rounded to the nearest tenth if needed.

1. $(2,0)$ and $(5,4)$
2. ( $-4,-3$ ) and $(2,5)$
3. $(-4,-3)$ and $(8,2)$
4. $(-7,-3)$ and $(8,5)$
5. ( $-1,4$ ) and ( 2,0 )
6. $(-1,3)$ and $(5,-5)$
7. $(1,-4)$ and $(6,8)$
8. $(-8,-2)$ and $(7,6)$
9. $(-3,-5)$ and $(0,1)$
10. (-1, -2) and (-3, 4)
11. $(3,-1)$ and (1, 7)
12. (-4, -5) and (7, 4)

## Use the Midpoint Formula

In the following exercises, (a) find the midpoint of the line segments whose endpoints are given and (b) plot the endpoints and the midpoint on a rectangular coordinate system.
13. $(0,-5)$ and $(4,-3)$
14. (-2, -6) and (6, -2)
15. $(3,-1)$ and $(4,-2)$
16. (-3, -3) and (6, -1)

## Write the Equation of a Circle in Standard Form

In the following exercises, write the standard form of the equation of the circle with the given radius and center ( 0,0 ).
17. Radius: 7
18. Radius: 9
19. Radius: $\sqrt{2}$
20. Radius: $\sqrt{5}$

In the following exercises, write the standard form of the equation of the circle with the given radius and center
21. Radius: 1, center: $(3,5)$
22. Radius: 10 , center: $(-2,6)$
23. Radius: 2.5 , center: $(1.5,-3.5)$
24. Radius: 1.5 , center: $(-5.5,-6.5)$

For the following exercises, write the standard form of the equation of the circle with the given center with point on the circle.


#### Abstract

25. Center $(3,-2)$ with point $(3,6)$


26. Center $(6,-6)$ with point $(2,-3)$
27. Center $(4,4)$ with point $(2,2)$
28. Center $(-5,6)$ with point $(-2,3)$

## Graph a Circle

In the following exercises, (a) find the center and radius, then (b) graph each circle.
29. $(x+5)^{2}+(y+3)^{2}=1$
30. $(x-2)^{2}+(y-3)^{2}=9$
31. $(x-4)^{2}+(y+2)^{2}=16$
32. $(x+2)^{2}+(y-5)^{2}=4$
33. $x^{2}+(y+2)^{2}=25$
34. $(x-1)^{2}+y^{2}=36$
35.
$(x-1.5)^{2}+(y+2.5)^{2}=0.25$
36. $(x-1)^{2}+(y-3)^{2}=\frac{9}{4}$
37. $x^{2}+y^{2}=64$
38. $x^{2}+y^{2}=49$
39. $2 x^{2}+2 y^{2}=8$
40. $6 x^{2}+6 y^{2}=216$

In the following exercises, (a) identify the center and radius and (b) graph.
41. $x^{2}+y^{2}+2 x+6 y+9=0$
42. $x^{2}+y^{2}-6 x-8 y=0$
43. $x^{2}+y^{2}-4 x+10 y-7=0$
44.
45. $x^{2}+y^{2}+6 y+5=0$
46. $x^{2}+y^{2}-10 y=0$
$x^{2}+y^{2}+12 x-14 y+21=0$
47. $x^{2}+y^{2}+4 x=0$
48. $x^{2}+y^{2}-14 x+13=0$

## Writing Exercises

49. Explain the relationship between the distance formula and the equation of a circle.
50. In your own words, state the definition of a circle.
51. Is a circle a function? Explain why or why not.
52. In your own words, explain the steps you would take to change the general form of the equation of a circle to the standard form.

## Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

| I can... | Confidently | With some <br> help | No-I don't <br> get it! |
| :--- | :--- | :--- | :--- |
| use the distance formula. |  |  |  |
| use the midpoint formula. |  |  |  |
| write the equation of a circle in standard form. |  |  |  |
| graph a circle. |  |  |  |

(b) If most of your checks were:
...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.
...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?
...no - I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.
247. (1, -4) and (5, -5)

Use the Midpoint Formula
In the following exercises, find the midpoint of the line segments whose endpoints are given.
248. (-2, -6) and ( $-4,-2$ )
249. $(3,7)$ and $(5,1)$
250. ( $-8,-10$ ) and (9, 5)
251. (-3, 2) and (6, -9)

Write the Equation of a Circle in Standard Form
In the following exercises, write the standard form of the equation of the circle with the given information.
252. radius is 15 and center is $(0,0)$
253. radius is $\sqrt{7}$ and center is $(0,0)$
256. center is $(3,6)$ and a point on the circle is $(3,-2)$
254. radius is 9 and center is $(-3,5)$
257. center is $(2,2)$ and a point on the circle is $(4,4)$
255. radius is 7 and center is $(-2,-5)$

## Graph a Circle

In the following exercises, (a) find the center and radius, then (b) graph each circle.
258. $2 x^{2}+2 y^{2}=450$
259. $3 x^{2}+3 y^{2}=432$
261. $(x+2)^{2}+(y+5)^{2}=49$
262.
$x^{2}+y^{2}-6 x-12 y-19=0$
260. $(x+3)^{2}+(y-5)^{2}=81$
263. $x^{2}+y^{2}-4 y-60=0$

### 11.2 Parabolas

Graph Vertical Parabolas
In the following exercises, graph each equation by using its properties.
264. $y=x^{2}+4 x-3$
265. $y=2 x^{2}+10 x+7$
266. $y=-6 x^{2}+12 x-1$
267. $y=-x^{2}+10 x$

In the following exercises, (a) write the equation in standard form, then (b) use properties of the standard form to graph the equation.
268. $y=x^{2}+4 x+7$
269. $y=2 x^{2}-4 x-2$
270. $y=-3 x^{2}-18 x-29$
271. $y=-x^{2}+12 x-35$

## Graph Horizontal Parabolas

In the following exercises, graph each equation by using its properties.
272. $x=2 y^{2}$
273. $x=2 y^{2}+4 y+6$
274. $x=-y^{2}+2 y-4$
275. $x=-3 y^{2}$

## PRACTICE TEST

In the following exercises, find the distance between the points and the midpoint of the line segment with the given endpoints. Round to the nearest tenth as needed.
327. (-4, -3) and (-10, -11)
328. $(6,8)$ and $(-5,-3)$

In the following exercises, write the standard form of the equation of the circle with the given information.
329. radius is 11 and center is $(0,0)$
331. center is $(-2,3)$ and a point on the circle is $(2,-3)$
330. radius is 12 and center is $(10,-2)$
332. Find the equation of the ellipse shown in the graph.


In the following exercises, (a) identify the type of graph of each equation as a circle, parabola, ellipse, or hyperbola, and bb graph the equation.
333. $4 x^{2}+49 y^{2}=196$
334. $y=3(x-2)^{2}-2$
335. $3 x^{2}+3 y^{2}=27$
336. $\frac{y^{2}}{100}-\frac{x^{2}}{36}=1$
337. $\frac{x^{2}}{16}+\frac{y^{2}}{81}=1$
338. $x=2 y^{2}+10 y+7$
339. $64 x^{2}-9 y^{2}=576$

In the following exercises, (a) identify the type of graph of each equation as a circle, parabola, ellipse, or hyperbola, (b) write the equation in standard form, and (c) graph the equation.
340. $25 x^{2}+64 y^{2}+200 x-256 y-944=0$
341. $x^{2}+y^{2}+10 x+6 y+30=0$
342. $x=-y^{2}+2 y-4$
343. $9 x^{2}-25 y^{2}-36 x-50 y-214=0$
344. $y=x^{2}+6 x+8$
345. Solve the nonlinear system of equations by graphing:

$$
\left\{\begin{array}{l}
3 y^{2}-x=0 \\
y=-2 x-1
\end{array}\right.
$$

346. Solve the nonlinear system of equations using substitution:
347. Solve the nonlinear system of equations using elimination:
$\left\{\begin{array}{l}x^{2}+y^{2}=8 \\ y=-x-4\end{array}\right.$

$$
\left\{\begin{array}{l}
x^{2}+9 y^{2}=9 \\
2 x^{2}-9 y^{2}=18
\end{array}\right.
$$

